Introduction to Modelling Relationships and Graphs

Content Discussion and Activities

PHYS 104L

1 Goal

The goal of this week's activities is to provide a foundational understanding and comfort in modeling and testing relationships between different physical quantities. The focus will be on developing graphical techniques for testing expected relationships between different physical quantities.

2 Introduction

We have already looked at the measurement process and collecting data by measuring physical quantities. We have discussed the details of performing calculations and statistical analysis with the data to determine results for other quantities and keeping track of uncertainties. One reason for wanting to collect data is to try and understand how one quantity may depend on changes in a second quantity. For example, what if we are keeping track of the distance a person can long jump. There are several different factors we might think the distance a person can long jump could depend on; running speed, take-off angle, type of shoes, running surface, body position in the air…. We could imagine trying to collect data to try and quantify and determine just how the long jump distance is affected by these other possible **variables**. To do so in a scientific manner, we would want to keep all the possible variables constant except for one which we would choose to vary. For example, we might keep all the above variables the same except for take-off angle which we would have the jumper vary from jump to jump. We would measure the value for take-off angle for each jump and then also measure the resulting long jump distance. Once we had several sets of data, different take-off angles with corresponding long jump distances, we could then try and determine the relationship between the two quantities – a formula or equation for how the long jump distance depended on the take-off angle, or test a predicted relationship. We could follow a similar process for how long jump distance depended on any of the other possible variables as well. It is the method for using the acquired data to test or determine a relationship that we will cover during this week and next week's activities.

We have already seen how to construct a graph of data and how to determine the coefficients for the equation of the best fit line to a set of data using a linear regression calculation. Today, we will build on and expand those skills to see how we can collect data and use graphical techniques to test expected relationships between different quantities. Graphical techniques aren't the only methods we could use, however, they are a particularly convenient method and offer the added benefit of being able to visually inspect the data over a range of values to check for both precision and accuracy. There are two different types of situations we could find ourselves in regarding modeling relationships and data, and the difference is whether we already have an expected or predicted relationship between two quantities or not. If we do have an expected relationship, then we would like to become familiar with a method for collecting data and using that data to **test** our expected relationship and see if it is consistent with the actual data. If we do not already have an expected relationship, we would like to become familiar with a method for collecting data and using that data to **determine** a possible relationship between the measured quantities. Using data to either determine or test an expected relationship between quantities are fundamental aspects of the scientific method and the graphical techniques we will cover are typical ways of carrying out that work. In today's activities, we will focus on testing already existing expected relationships while next week's activities will focus on determining actual possible relationships from collected data.

3A Discussion - Modeling Relationships with Equations

In much of the work we've already done this semester, we have used equations involving physical quantities. We may not have emphasized it before, but these equations really are models describing the relationship between the different quantities involved in the equation. In the density lab, we saw that the mass of an object was equal to the density of the material it was made out of multiplied by the volume of space occupied by that material: mass = density x volume. This equation described the relationship between those three quantities. Given some data, we can test to see if that expected equation really does describe the way the involved quantities are related.

3B Activity – Modeling Relationships with Equations

1.) The table below gives data (fake) for two quantities X and Y. What if you are told that the expected relationship between X and Y is that

$$Y = 7 * X.$$

Take a few minutes with your partner to try and determine whether or not this data is consistent with that relationship or not. Describe what you did, show whatever work you do, summarize your reasoning, and explicitly state, on your activity sheet in the correct place, whether you believe the data is consistent with the given relationship or not.

X (m)	Y (sec)
3	21
7	35
11	68
16	104
22	138
28	150

2.) A second person comes forward and says they were expecting that the relationship between X and Y should be described as

$Y = 6 * X + 12$.

Take a few minutes with your partner to try and determine whether or not you believe the same data is consistent with that relationship or not. Describe what you did, show whatever work you do, summarize your reasoning, and explicitly state, on your activity sheet in the correct place, whether you believe the data is consistent with the given relationship or not.

4A Discussion – Using Graphs and Linear Regression to Test Relationships

We need a method by which we can compare measured data with an expected relationship and definitively determine whether they are in agreement with each other or not. We want to do better than just be able to say "well, they look pretty close." To do so, we will build on what we have already studied regarding graphs and linear regression calculations. Recall, we could graph data and then perform a linear regression calculation to determine the slope and y-intercept of the best fit line to the data. The best fit line is the line which comes closest to going through the actual data points. Based on how much those data points scatter about the best fit line, the linear regression calculation also allows us to determine the uncertainty in the slope and y-intercept of the best fit line. If our model makes a prediction for what we would expect the values for the slope and the y-intercept of the graph to be, we would then have a very convenient way to graphically test a model regarding the relationship between different physical quantities. The general process would be

1.) Collect data for the two quantities, varying one then measuring the other, holding all other quantities constant.
2.) Use the theoretical model/relationship to identify how one can graph the data such that we would expect the graph to be linear.
3.) Make the graph and perform a linear regression calculation to determine the best estimate and uncertainty for the slope and y-intercept of the best fit line consistent with the data.
4.) Use the theoretical model/relationship to determine the EXPECTED values for the slope and y-intercept of the graph.
5.) Check to see if the expected values are in agreement with the results from the data and linear regression analysis.

If the expected values are within the experimental ranges, then we can say the data is consistent with the expected model. If one or both of the values are not in agreement, we have evidence that something is wrong; either there are systematic errors affecting our data or the model is incorrect.

4B Activity – Using Graphs and Linear Regression to Test Relationships

1.) Go back to the data from Activity 3B and use Excel to create a graph of Y vs X. Perform a linear regression calculation, using Excel, to determine ranges for the slope and y-intercept of the best fit line. Record both the result and range, properly rounded in the space provided on your activity sheet.
2.) On the graph, complete 4-line summaries to compare the experimental results for the slope and y-intercept to the Expected values as predicted by the 1st person's model; Y = 7*X.
3.) On the same graph, also complete 4-line summaries to compare the experimental results for the slope and y-intercept to the Expected values as predicted by the 2nd person's model: Y = 6*X + 12.
4.) Print a copy of the graph to turn in with your activity sheet. On a single page there should be a properly labeled graph, the raw linear regression coefficients, and four 4-line summaries, two to test person #1's model and two to test person #2's model.
5.) In the space on the activity sheet, state whether each person's model is consistent with the data and explain what you have based this answer on.

5A Discussion – Using a Theoretical Equation to Predict Graph Results.

In the prior example, we didn't have to do a lot of work to figure out what slope and y-intercepts the models predicted we should get. In addition, we didn't have to even think about what graph to make to test those models; the equations were already in the form of the equation of a straight line; "y" = m "x" + b. What if the given theoretical equation wasn't already in the form the equation of a straight line? Can we still do a graphical analysis, with a linear regression calculation, to test data to see if it is in agreement or not? The answer is yes, however, we need to add an extra step of identifying just what graph we could make which we would expect to be linear.

What if the model we had been trying to test was that $Y = 5.2 * X^3 + 7$. If we have data for X and corresponding Y values, and if we made a graph of Y vs X values, we would not expect it to be a straight line. Rather, it would give a curved shape. This would happen because Y would not be expected to be LINEARLY related to (or proportional to) X values. Fortunately, we are not constrained to only being able to plot X or Y values on a graph; we can plot just about anything we want. We expect to get a linear graph anytime the quantities we are plotting, I'll call them "x" and "y", are expected to be related just as in the equation for a straight line. If we can read a theoretical relationship as "y" = m "x" + b, where "y" is the quantity we plot on the vertical axis and "x" is the quantity we plot on the horizontal axis, with m and b being constants, then the plot is expected to be linear and m is the expected value for the slope of the graph and b is the expected value for the y-intercept.

Let's continue to consider the proposed model of $Y = 5.2 * X^3 + 7$. What if we used data to make a plot of Y vs X? For that graph, Y would be "y" and X would be "x". We could then rewrite the model relationship as

$$“y” = 5.2 “x”^3 + 7$$

Notice how this does not look like the equation of a straight line, “y” = m “x” + b. For us to expect a straight line, both the “x” and the “y” must be raised to the first power and that is not the case for a graph of Y vs X. We could remedy this by making a different graph. Instead of plotting X values on the horizontal axis, we could plot X^3 values on the horizontal axis and still plot Y values on the vertical axis. This would be a graph of Y vs X^3.. In that case, we could now write the expected relationship, in terms of what quantities we are graphing, as

$$“y” = 5.2 “x” + 7$$

This **does** look like the equation of a straight line. The quantity we are plotting on the vertical axis is equal to some constant (5.2) times the quantity we are plotting on the horizontal axis plus another constant (7). We would expect this graph, the graph of Y vs X^3, to be linear. In addition, we would expect the slope of the best fit line to be 5.2 and we would expect the y-intercept of the best fit line to be 7. In this way, given a wide range of expected relationships between different quantities, we can still identify some type of graph we could make which would be expected to be linear and identify the expected values for the slope and y-intercept of the best fit line.

5B Activity – Using a Theoretical Equation to Predict Graph Results.

For the rest of this activity, you will be considering a simple pendulum. The period (T) of a pendulum is the amount of time it takes to complete one full cycle of motion; to start at one endpoint, swing to the opposite side and then back again. The length (L) of a simple pendulum is the distance from the pivot point at the top, down to the center of the mass suspended from the pivot point. If the period is measured in Seconds and the length is measured in centimeters, the theoretical model describing how the period is related to the length of a pendulum is

$$T = 2\pi \sqrt{\frac{L}{g}}$$

where g is the acceleration due to gravity, 980 cm/s^2. This is the theoretical relationship we would like to test experimentally.

1.) At your lab table, there should be a pendulum already set up. You are to set the pendulum to a length somewhere between 10 and 100 cm. Use a meter stick to measure the length L. Record this value. Then use a stopwatch to measure the time for 20 full cycles to occur. Record this time measurement. Divide this value by 20 to determine your experimental value for the period of the pendulum. Record this value.
2.) Repeat this process for 5 other pendulum lengths spanning the range from 10 to 100 cm. In other words, one length should be close to 10 cm, one length should be close to 100 cm, and the other 4 should be spread out between those two values.

3.) Using these six sets of Length and Period data values, create a graph of T vs L. Does it appear to be linear? Based on the given relationship, would you expect it to be linear? Answer and explain on your activity sheet.

4.) Now you will create a new graph to consider, using the same raw data. This time, make a graph of Tvs $\sqrt{L}$. To do so, you will again use the measure T values for your y axis values, however, this time you will take the square root of each of your measured Length values and use these values for the horizontal axis values. The vertical axis values will be the same as for the first graph, however, the horizontal axis values will be different. Be careful, not only will the numbers you plot on the horizontal axis be different, but the units for those new numbers will be different as well since you had to do the same mathematical operations to the units that you did for the numbers. You also took the square root of the cm units too. Does this graph appear to be linear? Based on the given relationship, would you expect it to be linear? Answer and explain on your activity sheet.

5.) Perform a linear regression calculation to determine the slope and y-intercept of the best fit line. Compare those values with the theoretically expected values based on the given relationship between T and L. On one page, you should have the graph, the raw linear regression coefficients, and two four line summaries (one for the slope and one for the y-intercept). Print a copy of this page to attach to your activity sheet.

6.) On your activity sheet, show the details and explain how you determined the expected values for the slope and y-intercept of the best fit line for this graph.

7.) Now you will create a third graph to consider, using the same raw data. This time, make a graph of T^2 vs L. In other words, you will take your period values and square them. These are the values you will use for your "y" values and you will use the measured L values as your "x" values. Perform a linear regression calculation and compare the results for the slope and y-intercept of the best fit line with the theoretically expected ones based on the given relationship. Print a copy of this graph to attach to your activity sheet.

8.) On your activity sheet, show the details and explain how you determined the expected values for the slope and y-intercept of the best fit line for this graph.

Introduction to Modeling Relationships and Graphs

Activity Data Sheet

PHYS 104L

Activity 3B

Is the given data consistent with the expected model of Y = 7 * X? Yes No (Circle One)

Explain how you have determined your answer:

Is the given data consistent with the expected model of Y = 6 * X + 12? Yes No

Explain how you have determined your answer:

Activity 4B

Based on the Excel Graph and linear regression analysis, is the given data consistent with the expected model of Y = 7 * X? Yes No

Explain how you have determined your answer:

Based on the Excel Graph and linear regression analysis, is the given data consistent with the expected model of Y = 6 * X + 12? Yes No

Explain how you have determined your answer:

Activity 5B

Length	Time for 20 full cycles	Period
(cm)	(Sec)	(Sec)

Does the graph of T vs L appear to be linear? Based on the given relationship, would you expect it to be linear? Answer and explain here.

Does it appear to be linear? Yes No Did you expect it to be linear? Yes No

Explain:

Does the graph of T vs $\sqrt{L}$ appear to be linear? Based on the given relationship, would you expect it to be linear? Answer and explain here.

Does it appear to be linear? Yes No Did you expect it to be linear? Yes No

Explain:

Show how you determined the expected slope and y-intercept values for the T vs $\sqrt{L}$ graph:

Show how you determined the expected slope and y-intercept values for the T^2 vs L graph: