Competing Models, the Crater Lab

Graphs, Linear Regression, and Competing Models

PHYS 104L

1 Goal

The goal of this week's lab is to check your ability to collect data and use graphical analysis to determine which proposed model relationship is consistent with that data. You will also use graphical analysis to determine numerical values for constants for the best fit power law relationship to the data.

2 Introduction

In this lab, we will be studying the diameter of craters left in a container of sand when a steel ball impacts the sand after being dropped from some distance above. The higher the ball is released from, the more energy the ball has. We can calculate the amount of gravitational potential energy, in units of Joules, the ball has by knowing m, the mass of the ball in kg, g (the acceleration due to gravity g = 9.8 m/s^2), and the height the ball falls through, h, measured in units of meters. The initial potential energy is then

$$E = mgh$$

As the ball falls towards the sand, this potential energy is converted into an equal amount of kinetic energy stored in the motion of the ball. The ball collides with the sand and, if removed, leaves a crater of some diameter D. We might reasonably believe that the diameter of the resulting crater is related, in some way, to the amount of energy the ball has when it strikes the sand; more energy will result in a larger crater diameter. We will guess that this relationship is some sort of power law relationship such that

$$D = AE^n$$

where A is some constant. When you observe different sized craters on the surface of the moon, realize these craters correspond to different energied objects (astroids, meteorites,…) striking that surface and leaving different sized craters. If we knew values for A and n, given a value for energy E of an incident object, we could calculate the expected size of the resulting crater diameter. In this lab, we will try to identify the relationship between projectile energy, E in Joules, and crater diameter, D in centimeters.

There are different lines of reasoning which would predict different expected relationships between E and D. For instance, we might contend that the size of a crater should be linearly related to the incident energy E

$$D = AE^1$$

where A is a proportionality constant. If you doubled E, the size of the crater would be expected to double as well.

There are, however, other models for how craters are created when the incident energy of some impacting object is absorbed by the surface. According to theory, the kinetic energy of an impacting object is distributed among five processes: heating, comminution (the creation of new surface area), deformation, ejection of material, and seismic waves. If deformation is the most important process, then the volume V of the crater should scale linearly with the energy of the object. Since the volume V is proportional to the crater size to the third power, we would expect

$$D = AE^{1/3}$$

On the other hand, if material ejection absorbs most of the energy, then the kinetic energy is converted back into potential energy as material is lifted up out of the crater. In this case, we would expect

$$D = AE^{1/4}$$

The details behind these models are not the focus here. Instead, the main point is that there are several different competing models, based on different assumptions about what happens in this process, which result in different predicted models for how the crater diameter should depend on the projectile energy. We don't know which of these models, if any of them, are actually correct without studying the system and collecting data. It may turn out that none of these models are correct, implying some other process dominates the crater formation process.

Procedure

At your lab station, you should have a container, partially filled with sand, a steel ball, calipers, and a 2m meterstick. You will also need to make use of a consistent process for preparing the sand to collect data. You don't want the sand conditions to change while you collect data. As such, you will also need some object to use to level the sand prior to impact.

1.) To prepare the sand, rake through the sand with your hand, then level the sand surface with a flat object. You will want to perform this process prior to each time you drop the steel ball.
2.) Place the bottom end of the meterstick on the top of the sand surface. While one person holds the meterstick, the other partner will hold the ball 60 cm above the sand and away from the meterstick so that the crater will not be affected by the meterstick. Release the ball from rest and record the distance from the bottom of the ball, before it was released, to the sand surface (in this case it should be 60 cm).
3.) Use a small magnet to remove the ball from the sand without disturbing the crater. Use the Vernier caliper to measure the diameter of the resulting crater. Measure from the highest point on one side of the crater to the highest point on the opposite side of the crater.

4.) Repeat steps 1-3 to make 4 crater diameter measurements for drops from a height of 60 cm.
5.) Repeat steps 1-4 for 7 other heights, spread out between a minimum of 30 cm and a maximum of 180 cm.
6.) Measure and record the mass of the ball using the mass balance.

Calculations

1.) Open an Excel spreadsheet and set up a table with one row for each drop height. For the first column, list the drop height. In the next four columns, list each of the measured crater diameters. In the sixth column, calculate the average crater diameter.

2.) Create a seventh column where you calculate the energy E, in Joules, of the dropped ball.

3.) Create an eighth column where you calculate the natural log (ln) of the energy E and a ninth column where you calculate the natural log of the average crater diameter.

4.) Create a graph of ln(D) vs ln(E). Perform a linear regression calculation to determine the slope and y-intercept result and range for the best fit line. Use these results to determine what you think the best value for n is in a power law relationship between D and E. List this value and explain your reasoning in the provided space on the Data and Result sheet.

5.) Create a second graph of D vs E^n using your determined value for n. Perform a linear regression calculation to determine the slope and y-intercept of the best fit line. List these results and ranges, in a two line summary format, on the same page with the graph and pasted raw regression coefficients. Use these results to write out your determined best fit relationship between D and E. This should be an equation with D, E, and constants as determined from your second graph. Those constants should have uncertainties listed and units as well. Write this relationship out at the bottom of the page with your second graph.

6.) Print your Excel spreadsheet and each graph page to attach to your data sheet to turn in.

Lab Questions

1.) Which of the three listed models are consistent with your experimental data for energy and corresponding crater diameter? Which models, if any, would your data rule out?

2.) For your first graph, was the y-intercept of the best fit line in agreement with zero? Would you expect it to be? Explain why or why not.

3.) For your second graph, was the y-intercept of the best fit line in agreement with zero? Would you expect it to be? Explain why or why not.

4.) If you had measured the crater diameter in units of inches instead of centimeters, would you expect to get a different value for n in your determined power law relationship? Would you expect to get a different value for A in your determined power law relationship? Explain both of your answers.

5.) Pick one height from which you dropped the ball, and look at the variation among your measured crater diameters for that height. Now look at the average crater diameter for the next height (either up or down from the first one you looked at). How does the variation for the first height compare with the difference between the average crater diameter values for the successive heights? Based on this, do you think it would have been advisable to make more than four measurements at each drop height, or not?

6.) Assume you followed the same technique, using the same equipment, and dropped the ball from a height of 300 cm. Based on your results, what would you predict the diameter of the resulting crater to be? Show your work to defend your answer. We are expecting some calculations, not just a guess!

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Data Sheet / Report

PHYS 104L

Ball Mass (g) ________________ Ball Mass (kg) __________________

Drop Height (cm)	Crater Diameter Trial 1 (cm)	Crater Diameter Trial 2 (cm)	Crater Diameter Trial 3 (cm)	Crater Diameter Trial 4 (cm)

Determined value for n and reasoning to support it: