Waves on a String

1 Object

To determine the mass per unit length of various strings.

2 Apparatus

Assorted weights, clamps, electric oscillator, meter stick, electronic balance, poles, pulley, scissors, various strings.

3 Theory

Whenever a disturbance passes through an elastic medium, the disturbance will be transmitted from one area of the medium to another due to its tendency to restore an equilibrium situation. This disturbance is called a traveling wave. If the material is reasonably elastic and if friction and air resistance effects are minimal, then the shape of the wave will remain largely unchanged. Such is the case with the strings used in this experiment.

It is found that the speed of transmission of this particular wave depends on two quantities: the mass per unit length (density) of the string and the tension applied to the string. The relationship can be shown as follows:

Consider a small segment of a wave on a string, ΔL, in Figure 1, (the wave is assumed to be a transverse wave, *i.e.*, the particle motion is perpendicular to the wave motion).

If this segment is small enough, it can be considered to be an arc of a circle. Let the origin of this circle be the point O and its radius R. On either end of the string there will be a force due to the tension in the string and both forces will be tangential as shown. Because of the symmetry involved, they will be of equal strength and $T_1 = T_2$. Also, because of the symmetry, and with a

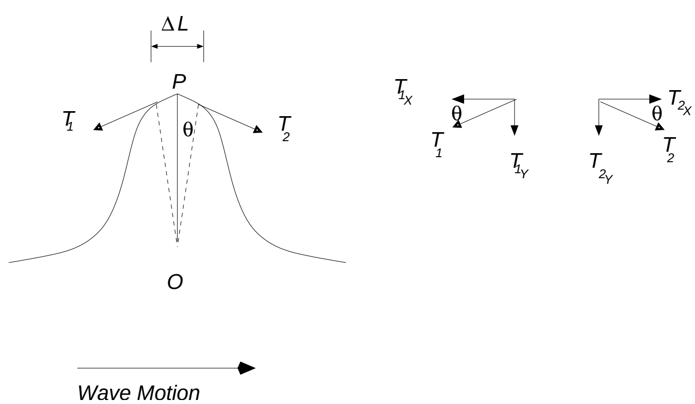


Figure 1: A displacement in a string as a wave passes by.

proper choice of coordinate system, OP vertical, the x components of the two tensions will cancel and their y components will add to $2T_y$. Assume further that this coordinate system moves along with the wave velocity v. With such a choice the segment will appear to move in a circle as the wave passes through; and then the net force on it, $2T_y$, will be a centripetal force. That is

$$F_c = \frac{mv^2}{R} = 2T_{1y}$$

For small ΔL (small θ) one has

$$T_{1y} \simeq T\sin\theta \simeq T\theta$$

If one uses the mass per unit length, μ, of the segment instead of its mass, one can use

$$m = \mu\,\Delta L$$

With this information, the centripetal force equation becomes

$$F_c = \frac{\mu(\Delta L)v^2}{R} = 2T\theta$$

or

$$v = \sqrt{\frac{2TR\theta}{\mu\Delta L}}$$

One further substitution from Figure 1 using the relation between angle and arc length, $2R\theta = \Delta L$, gives

$$v = \sqrt{\frac{T\Delta L}{\mu\Delta L}} = \sqrt{\frac{T}{\mu}}$$

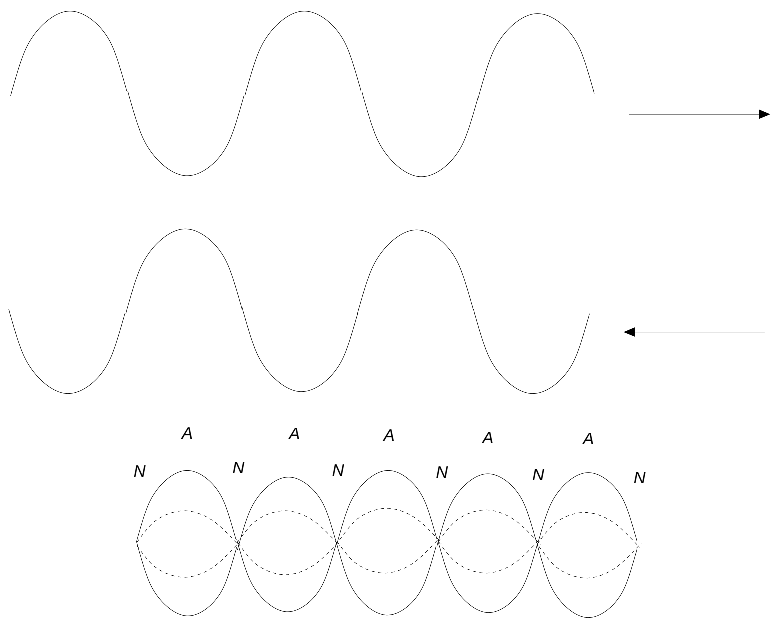
which relates wave velocity to tension and mass per unit length. It is rather difficult to see a traveling wave because of its small amplitude and relatively large velocity. If, however, the traveling wave can be transformed into a standing wave, observation is greatly simplified. This can be done by allowing the traveling wave to collide with a fixed object, the pulley. The traveling wave reflects back down the string after suffering a change of phase of 180°. Thus, one now has two waves on the string: These two waves will interfere, in general only partially constructively. But if the tension on the string is properly adjusted so that peaks and troughs of the two waves coincide, one has totally constructive interference and a resonance situation. In such a situation, the string will exhibit fixed nodes and anti-nodes, which will appear as follows: The nodes, N, will be dead spots on the string while the anti-nodes, A, will be areas of maximum displacement. If such an adjustment has been made, the wave form can easily be seen and measured, since the distance between any two adjacent nodes is half a wavelength.

In the current experiment, one wishes to determine the mass per unit length of various strings. Solving the previous equation for that quantity gives

$$\mu = \frac{T}{v^2}$$

The wave velocity can be determined if the wavelength and frequency are known and is

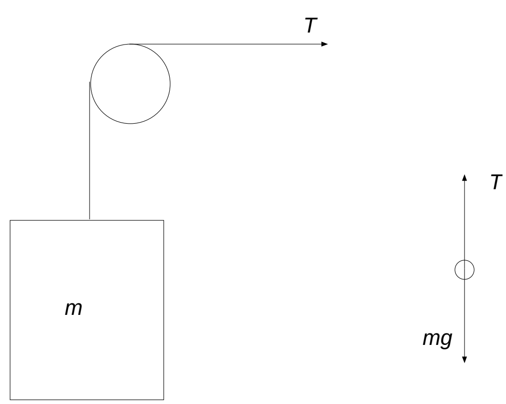
$$v = \lambda f$$



Combining this equation with the previous equation gives

$$\mu = \frac{T}{\lambda^2 f^2}$$

Lastly, the tension in the string is produced by weights attached to it at the pulley end. Applying



Newton's second law to the end of the string, one has $T = mg$, since the weights have no net force acting on them. This tension is assumed to be equally distributed throughout the string. Finally, one has arrived at a useful equation for experimentally determining the mass per unit length of a string

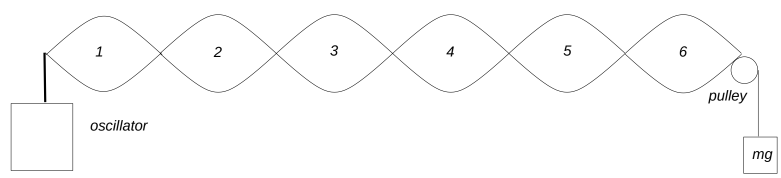
$$\mu = \frac{mg}{\lambda^2 f^2} \tag{1}$$

where m is the mass attached to the end of the string in grams, g is the acceleration due to gravity ($980.7\, cm/s^2$), λ is the wavelength of the wave on the string in cm, and f is the frequency in Hz .

4 Procedure

1. PLEASE NOTE – if you are using the variable frequency oscillators, DO NOT PUT MORE THAN 350 g ON THE END OF THE STRING. If you are using the fixed frequency oscillators, this is not an issue. Set up the apparatus as shown. Adjust the weights so that a standing wave is obtained. This occurs when the nodes (dead spots) on the string do not move and the anti nodes have maximum displacement. Record the frequency of the oscillator. Record the mass on the end of the string, the distance between any two adjacent nodes, and the number of sections on the string (in the diagram this is six). The distance between nodes is a half wavelength. Then, keeping the frequency fixed, change the mass until another standing wave (a different number of wave sections) is obtained.

   Repeat this procedure until five different standing waves are obtained. In each case, record the mass attached to the string and the distance between two adjacent nodes and the number of wave sections.

2. Attach the smallest mass used in procedure 1 to the string and mark two points on the string that are one meter apart. Then attach the largest mass used and measure the distance between the two marks. Cut the string at these two points and weigh it on the electronic balance.

3. Repeat parts one and two with the other strings.

5 Calculations

1. Using Equation 1, calculate the mass per unit length in each trial for each string. If you are using the fixed frequency apparatus, the frequency is 120 Hz.

2. For each string calculate the mean and error for the mass per unit length.

3. Using the general formula for the mass per unit length

$$\mu = \frac{m}{L}$$

   calculate μ using the data from procedures 2 and 3 for the balance data. There will be two results, one for each of the two lengths of the string between the marks. These two results represent the accepted range of values for the mass per unit length of a string. Using a four line summary, compare your results from the averages to this range. This should be done for each of the strings.

4. Equation 1 can be rewritten as

$$m = \frac{\mu}{g} f^2 \lambda^2 \tag{2}$$

If you find the best fit line for this equation using m as the y coordinate and λ^2 as the x coordinate you can compare it to the equation for the line

$$y = ax + b$$

where a is the slope and b is the y intercept. You would expect to find $a = \frac{\mu}{g} f^2$ and $b = 0$. Does your best fit agree with these expectations? For each string, use the low and high μ values you obtained with the scale data to calculate a high and low range for your slope. On the graph compare your slope to this using this range as a theoretical range. That is, a four line summary where the theory value is this range. State whether you get agreement.

6 Questions

1. Why do the thinner strings require less attached mass than the thicker ones to produce the same number of wave sections?
2. How would the experiment differ if the oscillator frequency were $800\, Hz$?
3. For the least squares fit in calculation 4, why should b=0?