

Ohm's Law

1 Object

To study resistors, Ohm's law, linear behavior, and non-linear behavior.

2 Apparatus

Resistors, power supply, meters, wires, and alligator clips.

3 Theory

A resistor is a circuit element which dissipates electrical energy in the form of heat. All conductors (metals) have resistance, but the material used in most resistors is carbon-glass. Resistance manifests itself at the microscopic level as collisions between conduction electrons (those actually involved in carrying the current) and the atoms making up the resistive material. Resistance is actually determined by the geometry of the material, and the intrinsic property of a material which is the macroscopic manifestation of the collisions between the electrons and the atoms is called the *resistivity* of the material. The resistivity and the material geometry determine the resistance of the material, and the relationship for most geometries is

$$R = \frac{\rho l}{A} \quad (1)$$

where ρ is the resistivity of the material, l is the length over which the current flows, and A is the cross sectional area through which the current flows. These are shown below in figure 1.

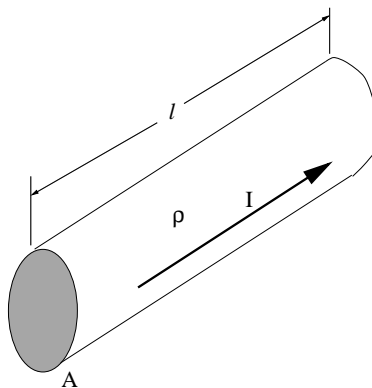


Figure 1: The geometry for the above equation.

Ohm's Law states the following:

$$R = \frac{V}{I} \quad (2)$$

where V is the voltage across the resistor in volts (V) and I is the current flowing through the resistor in amps (A). The units of resistance are then $V/A = \Omega$, called ohms. If the resistor is linear, then the ratio V/I is always the same and the resistance is a constant value. Then we may write

$$V = IR \tag{3}$$

which states that V and I are proportional with a constant of proportionality R . This is the linear form of Ohm's law, and the most famous form. All standard resistors follow this mathematical equation and are therefore technically *linear* resistors. Sometimes, for some materials or devices, this is not the case. Such devices are then termed *non-linear*.

When we build a circuit with resistors we can connect them in general in one of two ways: *in series* or *in parallel*. The way a resistor is sketched, and two resistors in series, are shown below. Putting two resistors in series makes the net (equivalent) resistance the sum of the resistors:



Figure 2: The schematic representation of a resistor (left) and two resistors in series (right).

$R_{eq} = R_1 + R_2$, or in general for many resistors

$$R_{eq} = R_1 + R_2 + R_3 + \dots + R_N \tag{4}$$

for a total of N resistors in series. For a parallel configuration, the resistors look like figure 3. The

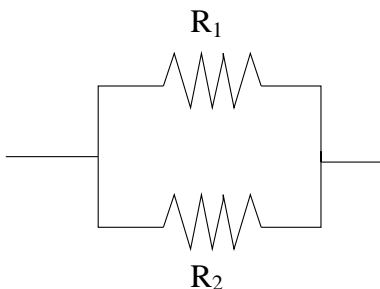


Figure 3: The schematic representation of two resistors in parallel.

net resistance for the parallel resistors is given by the following expression

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{5}$$

where R_{eq} will be less than either of the two resistances making up the parallel combination. For many resistors in parallel, the equation becomes

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots + \frac{1}{R_N} \tag{6}$$

and, once again, R_{eq} will be less than the smallest resistor in the combination.

Usually the circuit has a mixture of parallel and series resistors, and to find the equivalent resistance of the entire circuit one must add different sections in either series or parallel. For example, figure 4 shows such a circuit. The equivalent resistance between points A and B is found

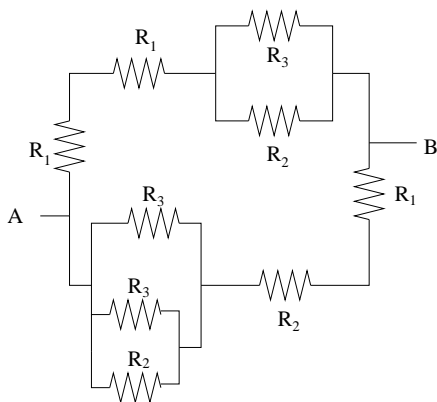


Figure 4: A sample circuit with series and parallel resistors.

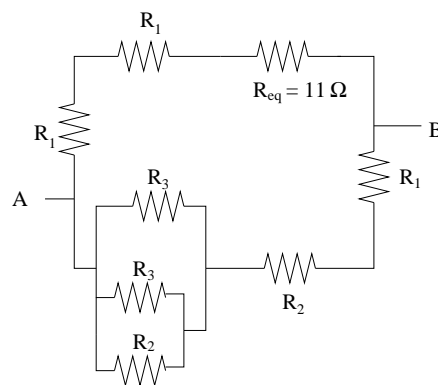


Figure 5: The same schematic but with R_{eq} in place.

using the above equations for adding resistors. Let $R_1 = 10 \Omega$, $R_2 = 20 \Omega$, and $R_3 = 25 \Omega$. The upper right pair is in parallel and can be added as such, yielding the equivalent resistance of the pair in its place: $1/R_{eq} = 1/20 \Omega + 1/25 \Omega$. This gives a value of $R_{eq} = 11 \Omega$, and this equivalent resistance can then replace the pair in figure 5.

Similarly, the lower left three resistors can be added in parallel and replaced by their equivalent resistance: $1/R_{eq} = 1/R_3 + 1/R_3 + 1/R_2$. Plugging in the values for R_2 and R_3 gives $R_{eq} = 7.7 \Omega$, with the schematic as shown in figure 6. At this point, there are resistors (between points A and

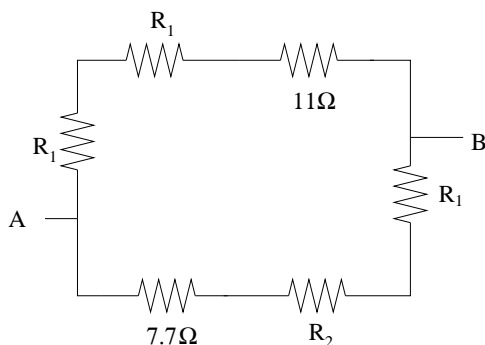


Figure 6: The schematic now with R_{eq} in place for the three parallel resistors.

B) in series on both the top path and the bottom path. We can calculate the equivalent resistance for the top ($R_{eq} = R_1 + R_1 + 11 \Omega = 31 \Omega$) and the bottom ($R_{eq} = 7.7 \Omega + R_1 + R_2 = 37.7 \Omega$) and replace the series resistances with these. The schematic now looks like figure 7. Finally, figure 8 shows two resistors in parallel, which can be added as such to yield the overall equivalent resistance for the entire circuit: $1/R_{eq} = 1/31 \Omega + 1/37.7 \Omega$, which gives $R_{eq} = 17 \Omega$. This is the amount of resistance between points A and B of the original circuit shown in figure 4, but in a simplified form.

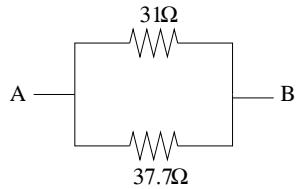


Figure 7: The schematic now with R_{eq} in place for the series resistors.

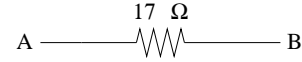


Figure 8: The equivalent circuit for the original circuit of figure 4

That is, these two circuits (figure 4 and figure 8) have the same net resistance between points A and B, even though one is much more complicated than the other. A battery connected between points A and B will supply the same current to either circuit.

3.1 Voltmeters and Ammeters

Voltmeters measure the potential difference *across* some circuit element or elements. The voltmeter is placed in parallel with the element and since it has a very high resistance it does not draw any current from the circuit and therefore does not affect the circuit. The reading on the meter is the voltage drop between the two points where the meter leads are placed in the circuit. This is depicted in figure 9. Ammeters read the current passing *through* a particular branch of a circuit

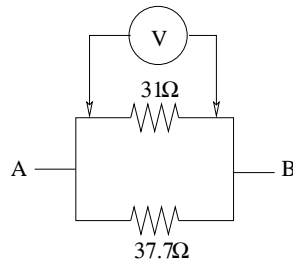


Figure 9: A voltmeter reading the potential difference across a resistor in a circuit.

(*i.e.* a wire). The ammeter must be wired directly into the circuit *in series* where one wants to know the current. The resistance of the ammeter is very low (essentially zero) and therefore does not affect the circuit. This is depicted in figure 10.

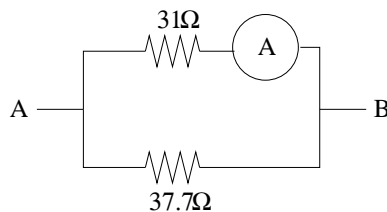


Figure 10: An ammeter reading the current through a wire in a circuit.

4 Procedure

4.1 Resistor

Choose a resistor and wire it up as shown in figure 11 with the ammeter in series and a variable voltage supply. You will vary the voltage on the supply and read the voltage drop across the resistor

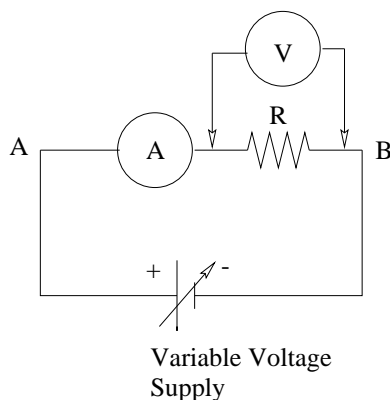


Figure 11: The circuit for part 1.

with the voltmeter. The corresponding value of the current should also be noted. Do this for eight different voltage values over a wide range. Repeat for each of the other three resistors. Make sure you know which resistor goes with which data for the remainder of the lab. Note the color bands of the resistors on your data sheet. Use a digital voltmeter on the Ω scale to find the value of each of your resistors – this is the real (theoretical) value.

4.2 Series

Wire two resistors in series and record their values. Hook up an ammeter in series as well as the voltage source. For three different voltages, measure the voltage across the pair and each individual resistor. Record the current as well for each voltage. Sketch your circuit, and repeat this for a different pair of resistors.

4.3 Parallel

Wire two resistors in parallel and record their values. Hook up the ammeter as shown in figure 12. For three different voltages, measure the current into the circuit as well as the voltage across the circuit, and then measure the voltage across each resistor. Sketch the circuit, and repeat for a different pair of resistors.

5 Calculations

5.1 Resistor

For each resistor, plot V versus I for your data. The resistance of each can be found by doing a linear regression of this data. These values should be compared to the theoretical R values you

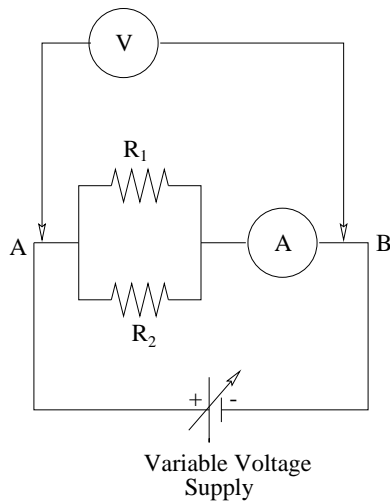


Figure 12: The parallel circuit.

obtained from the digital meter. You should also compare them to the values given by the color bands on the resistors – that is, four-line summaries treating the color band value \pm tolerance resistance as an experimental result.

5.2 Series

Calculate the total resistance of your series circuits using V and I . Using the equation for adding series resistances, calculate the equivalent resistance of the pair (using theoretical values). Compare your average equivalent resistance value \pm error in a four-line summary for each pair of resistors to what the theoretical values, when added in series, say the value should be. In addition, note how the two individual voltages compare to the total voltage across the pair. What is the relationship?

5.3 Parallel

Calculate the total resistance of the pair from V and I for each trial for a given pair of resistors. Find the average and associated error for the trials. Find the equivalent resistance using the theoretical resistances, and compare in a four line summary to your average \pm error. Calculate the I through each resistor using V and R . How do the two I 's compare to the total I ?

6 Questions

1. On the board is a table showing how to read the color bands on resistors. For your resistors, what are their R values based on these bands? What is the accuracy (tolerance)? How do the values compare to your values from the lab? (*i.e.* both theoretical and from the graphs in part 1)
2. Explain why when one puts two resistors in series the resistance increases while for the same two in parallel the resistance decreases.

3. We assumed your ammeter had very little resistance (it really is pretty small), but its R value $\neq 0$. Explain the effect your ammeter has on a circuit when it is placed in the circuit.
4. If you first measure V across R (position #1 of the figure) and use I from the ammeter, you get a resistance value from Ohm's law. If you then measure V as shown in position #2 of the figure and use I from the ammeter you will get a different R value. If you are trying to determine the resistance of the resistor in the circuit, which is correct? Explain.

