

Kirchhoff's Laws, Internal Resistance of a Battery, Oscilloscopes, RC Circuits

1 Object

To study Kirchhoff's Laws, internal resistance, oscilloscopes, and RC circuits.

2 Apparatus

Resistors, power supply, meters, wires, alligator clips, oscilloscope, capacitor, battery.

3 Theory

Kirchhoff's Laws for circuit analysis are:

1. the sum of the currents into a junction (a connection of at least three wires) equals the sum of currents leaving the junction

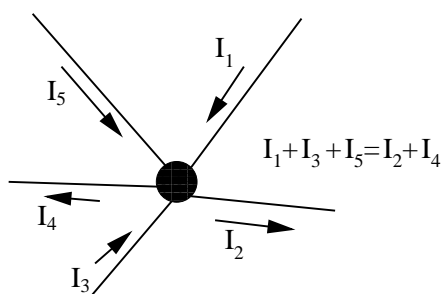


Figure 1: Currents entering and leaving a junction.

2. the algebraic sum of the voltage drops (negative) and gains (positive) in “walking” around a closed loop of a circuit is zero.

For example, in the simple loop depicted in figure 2 the resistor is simply hooked across a battery. One first must label all possible currents in the circuit, and here the only one is I . One starts the walk at the point indicated and goes around the loop as shown by the curved arrow. The resistance of wires connecting elements like resistors and batteries are ignored, so the first thing we encounter is the resistor. In going from left to right across this resistor, in the same direction as I , we experience a drop in voltage of IR , which we will write down with a *minus* sign:

$$-IR \dots \quad (1)$$

Continuing on, we next encounter the battery. Walking in the same clockwise sense, we go from the negative terminal to the positive terminal of the battery, which corresponds to an increase in

voltage equal to that of the battery's voltage V . Hence we can write

$$-IR + V = 0 \tag{2}$$

where the $= 0$ part comes from the fact that after we leave the battery we return to our starting point. In using Kirchoff's laws the goal is usually, but not always, to find the current(s) in the

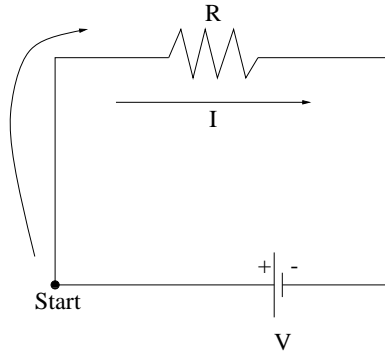


Figure 2: A simple circuit loop with I labeled, as well as the starting point and direction of the “walk.”

circuit.

3.1 Batteries

Batteries are sources of EMF (electro-motive force) – they supply electrical energy to circuits via chemical reactions. For our purposes, we will represent a battery as an ideal EMF source in series with a resistance r , the internal resistance of the battery. For good (new) batteries, r is small and has little effect on the circuit's behavior. For older batteries, r is larger and the effects are easily seen and measured. The circuit for our old batteries looks as in figure 3. The stuff inside the dotted line is how we represent the inside of the battery. The emf \mathcal{E} and the internal resistance r in series can then be used as elements in the circuit. Note that the actual battery terminals are labeled as (+) and (-). The battery is hooked up to some circuit, here labeled as a load resistance R_L . Now let's walk around the circuit and use Kirchoff's laws to analyze the circuit. Walking in a clockwise sense (same as the current) and starting at the negative terminal of the battery, we get:

$$\mathcal{E} - Ir - IR_L = 0 \tag{3}$$

But IR_L is equal to the voltage across the terminals of the battery while it supplies the current I . This is called the terminal potential difference, V_T . We can now write

$$\mathcal{E} - Ir - V_T = 0 \tag{4}$$

which can be solved for the internal resistance as

$$r = \frac{\mathcal{E} - V_T}{I} \tag{5}$$

So, if one measures the terminal potential difference (V_T) while the battery is supplying a current I , and then measures the \mathcal{E} of the battery when it is out of the circuit, one can calculate the internal resistance of the battery.

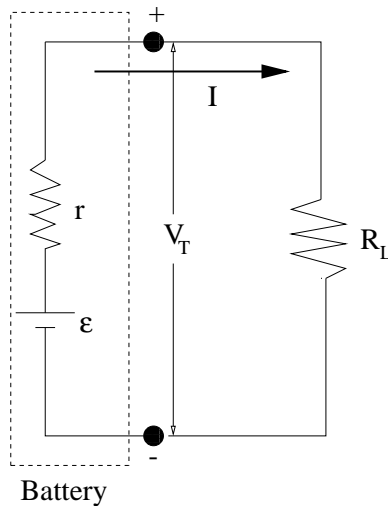


Figure 3: A battery represented by an EMF source \mathcal{E} and internal resistance r . It is hooked up to a circuit represented by the resistance R_L .

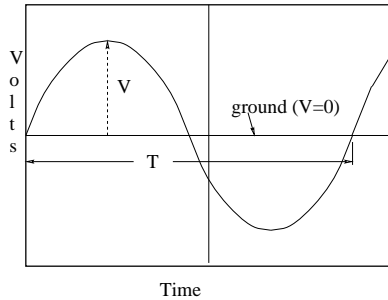


Figure 4: A sketch of the oscilloscope display with a sine wave displayed. The voltage and period are indicated by V and T , respectively.

3.2 Oscilloscope

An oscilloscope is one of the most useful electronics instruments available. It provides a visual means of determining what is happening in a circuit. The scope is composed of three main parts: Display, time base, and voltage scaler. The display is the small video-like window with the grid and axes on it. It has a beam that sweeps across from left to right at speeds which are selected with the **time base** knob. The time base knob adjusts the time the beam takes to cover 1 cm horizontally, and is in some number of seconds or fractions of a second. The **voltage scaler** allows an input voltage and affects the display vertically. With zero volts input, the sweep should stay centered at zero (the middle of the display). With a positive DC voltage input, the sweep should jump up above the center an amount proportional to the voltage input. That is, if one input $+5\text{ V}$ and the voltage scaler was set to 1 V/cm , then the sweep line should move up a distance of 5 cm from the ground level on the display. With an AC signal, the oscilloscope will display the time variation of the voltage input. If one hooks up the function generator with a sine wave output to the oscilloscope, proper selection of the time base and voltage scaler will display a sine wave. One

can then measure off of the screen the amplitude of the wave (the voltage of the sine wave) as well as the period of the wave. One does this by counting the number of grid blocks (*cm*) vertically from ground to the maximum and multiplying by the voltage scale, and by counting the number of blocks horizontally between peaks (or valleys) and multiplying by the time base scale, respectively. Oscilloscopes are most useful when one has a circuit with time varying voltage and current, because it lets one see the behavior of the circuit moment by moment. This makes it more useful than a digital meter, for instance.

3.3 RC Circuit

An RC circuit is a circuit with a resistance R and a capacitance C hooked up in series. This is shown in figure 5. An RC circuit is a very interesting circuit. If one first imagines a capacitor being

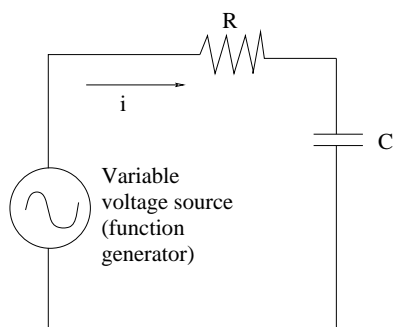


Figure 5: An RC circuit with a function generator supplying a time varying voltage.

charged by a battery (no resistance), the capacitor charges *instantly* to a value given by $Q = CV$, where C is the capacitance of the capacitor and V is the voltage placed across it. With a resistance in the circuit, the charging process takes longer due to the resistor's resisting the flow of charge between the plates of the capacitor. Most books have the following result for charging a capacitor through a resistor:

$$V(t) = V_o \left(1 - e^{-\frac{t}{RC}}\right) \quad (6)$$

Since all exponents must be unit-less, and t has units of s , then the product RC must have units of s as well. Therefore the product RC is called the *time constant* of the circuit, and is denoted by the Greek letter tau: $\tau = RC$. The value of the time constant will depend on what one uses for R and C . But what does the time constant tell us? It gives us a relative idea of how fast the capacitor will charge up. A bigger τ value means that the capacitor charges more slowly, and a smaller τ means that the capacitor charges faster. For a capacitor which is initially charged and discharges through a resistor, the equation is

$$V(t) = V_o e^{-\frac{t}{RC}} = V_o e^{-\frac{t}{\tau}} \quad (7)$$

This is depicted in figure 6, where the initial voltage across the capacitor is V_o , and then it discharges through a resistor R . The time for the voltage across the capacitor to drop to $1/e$ of V_o is the time constant τ . This is shown in the figure, and can be shown mathematically as well using equation 7. One sets $V(t) = V_o/e$, which becomes

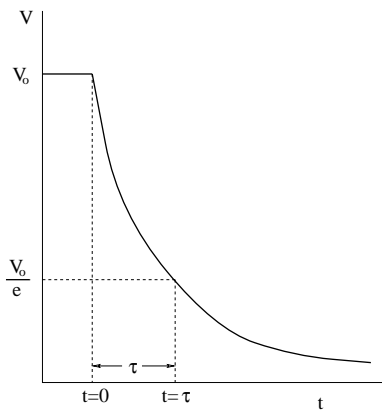


Figure 6: The graph showing the discharging of a capacitor from an initial voltage V_0 . The time for the voltage to drop to V_0/e is called τ , the time constant of the circuit.

$$\frac{V_0}{e} = V_0 e^{-\frac{t}{\tau}} \quad (8)$$

this leads to the condition that $t = \tau = RC$, meaning that τ is the time for the voltage to drop to $1/e$ of its initial value. This time will be longer if either R or C is larger – the circuit takes longer to discharge. This time will be shorter if either R or C is smaller – the circuit will discharge faster.

4 Procedure

4.1 Internal resistance of a Battery

1. Wire up your battery in series with an ammeter and a resistor as shown in figure 7

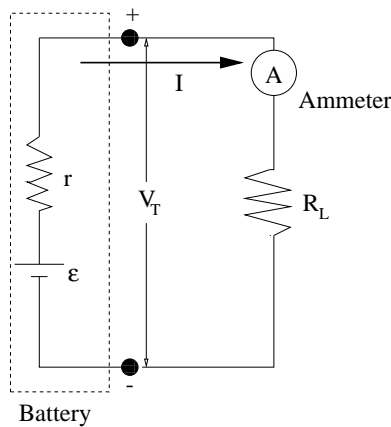


Figure 7: The circuit for finding the internal resistance of the battery.

2. For this configuration, record the current I and terminal potential difference V_T of the battery while it is supplying current.

3. Unhook the battery and measure the voltage at its terminals – this is \mathcal{E} .
4. Repeat the above for five different load resistors.

4.2 Oscilloscope

1. Turn on the oscilloscope and attach it to the function generator. Turn on the function generator and have it output a sine wave. Adjust the oscilloscope time base and vertical scale until you have a nice sine wave pictured.
2. Measure the voltage and period of the wave on the screen as described above, including what the voltage scale and time base are. Write down what the frequency knob of the function generator says the frequency is, and sketch the wave, labeling the amplitude and period.
3. Adjust the frequency knob to a different frequency and then adjust the oscilloscope time base until a nice sine wave is pictured. Provide all of the same information for this as for the first.
4. Do this all again for one other frequency that is much different than either of the first two.
5. Set the function generator to output a square wave. Vary the frequency and voltage and note what happens on the display. For three very different frequencies, provide the same info as for the sin waves.

4.3 RC Circuit

1. Wire up an RC circuit as shown above in figure 5. Set the function generator to output a square wave. Hook up the oscilloscope across the capacitor to view the voltage across it as a function of time.
2. Adjust the oscilloscope so that you can view the variation clearly.
3. Sketch what you see, and indicate what the voltage is as well as the period.
4. Vary the frequency of the function generator to a low value of frequency – note how the waveform changes. Sketch again and label V and T .
5. Make the frequency very high, and repeat the sketch and labels.
6. Estimate the reading error both vertically and horizontally for the scope. For your high frequency data, how much could the voltage and period be off due to this reading error? Calculate.
7. Now pick a frequency where it appears that the voltage is leveling off – you’ll know what this means when you see it, or I’ll tell you. Note the voltage of the level part (horizontal), and calculate $1/e$ of this. Determine the time the voltage takes to drop to this value, as in figure 6.
8. Repeat this for three different resistors.

5 Calculations

5.1 Internal Resistance

1. Calculate the internal resistance of your battery for each of the different load resistors. How do they compare? Should they all be the same?
2. Take equation 4 and solve for V_T – that is, using the equation in symbolic form, re-arrange the equation. Compare to the equation for a line, assuming one was to plot V_T versus I . What should the slope and intercept be?
3. Plot V_T versus I and do a linear regression. From the slope and intercept find r and \mathcal{E} . How do they compare to your calculated values? Does the plot look like it should?

5.2 Oscilloscope

1. For each waveform you sketched, show the values of the time base and the voltage scale. Show your calculation for the period.
2. For each of the periods you found, calculate the corresponding frequency using $f = 1/T$. Compare these frequencies to those read off of the knob.
3. Make sure you do the above for the square wave as well.

5.3 RC Circuit

Calculate for your R and C combinations the various τ values for your circuits. Compare these values to what you got by reading the time off of the oscilloscope screen. Are they close? Why/why not? Which might be a better value?

6 Questions

1. Often when a car battery “dies” it will still read 12 V when removed from the car and checked with a voltmeter. What should one do to see if the battery really is dead?
2. When you adjust the knobs on the oscilloscope, the picture it shows changes. What effect does this have on the signal which is being input to the oscilloscope? Explain.
3. What might be a real life use for an RC circuit?
4. Estimate the reading error from the scope for a horizontal measurement. Using this, estimate (yes, doing math) the error in a time measurement. What can you now say about the time constant values you read from the scope?