Electrostatic Field Mapping

1 Object

To investigate the Electric field for various geometries of conductors under an applied potential difference and to visualize the field.

2 Apparatus

Variable voltage supply, wires, conductive paper with various conductive geometries, voltmeter, transparency, pens.

3 Theory

Charges create electric fields in the region of space around them. One can discover if an electric field $\vec{E}$ exists at a certain point using a positive test charge $^+q_{test}$ and seeing if a force exists on it solely due to its charge. The direction of such a force is also the direction of the electric field at that point, and $\vec{E}$ is defined as

$$\vec{E} = \frac{\vec{F}}{^+q_{test}} \tag{1}$$

Associated with $\vec{E}$ fields is the electric potential difference ΔV. It is related to the electric field by

$$\Delta V = -\int_A^B \vec{E} \cdot d\vec{s} \tag{2}$$

where $\vec{E} \cdot d\vec{s} = Eds\cos\theta$ is the dot product of the $\vec{E}$ value at some point and a differential displacement $d\vec{s}$ at the same point. ΔV is the work one would do in moving a charge q from point A to B along some path divided by the charge q one is moving. This division by q makes ΔV independent of the charge which has been moved, and hence ΔV is really related/tied to the electric field.

Physicists, being essentially lazy people, generally use V to mean potential difference, as opposed to ΔV (it is less to write). To be correct, though, V is really $\Delta V = V_f - V_i$, where $V_i = 0$. So, V is really a potential difference with respect to the place where $V_i = 0$.

For a point charge, $E = \frac{kq}{r^2}$ and $V = \frac{kq}{r}$. Here, $V \to 0$ as $r \to \infty$, meaning V is really the potential difference between ∞ and the point in question (a distance r from the point charge). A sketch of $\vec{E}$ around a point charge is shown in figure 1. Since $V = V(r)$, spherical surfaces ($r = $ constant) are surfaces of constant potential ($V = $ constant). These surfaces are called *equipotential surfaces*, and are also shown as circles in figure 1. Note that the $\vec{E}$ lines intersect the constant V lines at right angles – this is true for *all* $\vec{E}$ and V geometries.

If one is dealing with a conductor, $\vec{E} = 0$ inside and the volume of the conductor is all at the same potential V as on its surface. Conductors are therefore called *equipotential volumes*. Since the $\vec{E}$ lines must intersect constant V lines at right angles, the $\vec{E}$ lines must leave the surface of conductors at right angles, too.
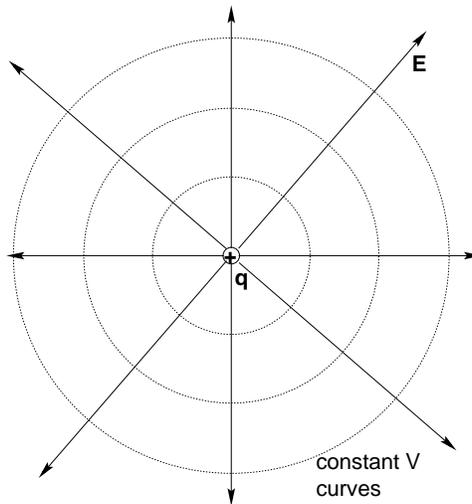
Figure 1: The electric field and equipotential lines for a positive point charge.

4 Procedure

4.1 Transparency

1. Select a piece of conducting paper from a folder and insert it on the board provided (not point charges). Swing the electrical contacts around so that they make contact on the silvery geometrical shape on the paper.

2. Make sure the power supply adjust knob is rotated counter clock wise and turn it on. Adjust its voltage value to between $15\,V$ and $20\,V$. Use the voltmeter to check the voltage between the $(+)$ and $(-)$ terminals of the power supply. This reading is better than the power supply's meter.

3. Now check the voltage reading between the silvery sections on the paper. Notice that the voltage reading changes depending on where you place the meter probes.

4. Choose a spot for the negative probe (on the negative silvery part) and keep it there forever. Now use the other probe (the $(+)$) to probe the voltage of non-silvery areas relative to the negative probe.

5. In general, the voltages vary with position. You should be able to find certain points which all have the *same* voltage value. These points should all be following each other, and they determine a line of constant potential (or an equipotential).

6. You need to map out various lines of constant potential onto a piece of transparency using transparency markers. First, trace the outline of the silvery geometry onto the transparency by laying the transparency over the black paper. Also mark the points of the square grid onto the transparency.

7. Remove the transparency, and probe for constant voltage lines. Use the grids to effectively transfer the probe position from the black paper over to the transparency.

8. Do this for as many voltages as you can in $2\,V$ steps

4.2 Graphically

1. Select a second sheet of different geometry (point charges) and hook up the electrical contacts as before.

2. This time you will record the potential reading from the meter for each of the points on the square grid on the black paper. That is, start at the bottom left of the grid and let this be the $x = 1$ and $y = 1$ point.

3. Now jump down in your reading to the calculations section for graphically.

5 Calculations

5.1 Transparency

The only calculation is to properly draw in $\vec{E}$ field lines once the equipotentials are sketched onto the transparencies. Draw in enough $\vec{E}$ lines to give the field shape in all areas of the transparency, and indicate the direction correctly as well. In your conclusion you should discuss where the field is larger and where it is smaller and why.

5.2 Graphically

1. Type the numbers into a grid of Excel spreadsheet cells that mimics the grid you are reading the voltage values from. So, if the grid on your paper is 12x16, then you should be filling cells on a 12x16 block in Excel.

2. Once typed in, highlight the entire block of cells. Then click on the *Insert* tab at the top, and then Other Charts, and then down at the bottom click on *All Chart Types*. This will cause a window to appear.

3. On this new window scroll down to the area titled **Surface** and click on the right-most icon (if you hover with the mouse, it should pop up with an info dialog that says *Wireframe Contour*). Then click on the *OK* button.

4. The contour plot should now appear. Click on one of the vertical axis labels (usually labeled *Series2*, for example), and then click on the *Format* tab at the top (under *Chart Tools*). Now, over on the top left, under the *File* tab, it says *Depth (Series) Axis* with a little down arrow next to the words. Click on the down arrow, which gives a drop down menu, and select from this *Vertical (Value) Axis*. The drop down menu disappears. Then, just below where the little arrow is, it says *Format Selection*. Click on this, and a new window should appear.

5. In this new window, under *Axis Options*, there is a *Major Unit* line. Change this from *Auto* to *Fixed* (click on the button next to *Fixed*). Change the value in the box from 2.0 to 1.0, and then click *Close*.

6   Questions

1. Why must an electric field line leave the surface of a conductor at right angles? Explain.

2. Does it appear that your electric field lines will/should leave the conducting regions perpendicularly? Why/why not?

3. What does it mean that the conducting areas on the paper get warm after some time?