The Current Balance

1 Object

To become familiar with the forces acting between two parallel, current carrying conductors. You will additionally determine an experimental value for μ_o, the permeability of free space.

2 Apparatus

The current balance apparatus, a high current voltage supply, two ammeters, laser, meter stick and tape measure.

3 Theory

The MKS system of units defines the ampere in terms of an existing force between two parallel conductors carrying current. The current balance may be used to make this measurement. The ampere is defined as follows: "one ampere is the unvarying current which, if present in each of two parallel conductors of infinite length and one meter apart in empty space, causes each conductor to experience a force of exactly $2 \times 10^{-7}\ N/m$." The current balance is primarily a device for measuring currents to a high degree of precision. The current to be measured is passed in opposite directions through two parallel horizontal bars which are connected in series. The lower bar is fixed; the upper bar is balanced a few millimeters above it by adjusting a counterpoise. The upper bar supports a small pan into which analytical weights are placed causing the upper bar to drop down toward the lower one.

When the current is turned on and increased, repulsion between the two bars causes the upper bar to rise above its equilibrium position. The position of the bar is observed by means of a mirror mounted on a base and a laser which shoots a beam of light on the mirror. Using the reflected beam, the weight which may be lifted due to the current is determined.

The force on a straight conductor of length ℓ carrying a current I placed in a uniform magnetic field B is

$$\vec{F}_{mag} = I\vec{\ell} \times \vec{B} \tag{1}$$

If the current is flowing in a direction perpendicular to the direction of the magnetic field B, then the magnitude of this magnetic force is given as

$$F_{mag} = \ell I B \tag{2}$$

From the Biot-Savart law, it can be shown that the magnetic field B associated with an infinitely long, straight conductor carrying a current I at a distance d away is

$$B = \frac{\mu_o I}{2\pi d} \tag{3}$$

Since each conductor lies in a magnetic field set up by the other, each will experience a force. The magnetic field B at the location of the top wire due to current flowing through the bottom wire may be found using equation3 when they are separated by a distance d. The force on a length ℓ of the top conductor, due to the magnetic field from the lower wire, is found by combination of equations 2 and 3 yielding

$$F_{mag} = \ell I_{upper} \frac{\mu_o I_{lower}}{2\pi d} \tag{4}$$

a force which is repulsive if the currents flow through the conductors in opposite directions and attractive if currents flow in the same direction. Note, since the conductors are connected in series, the currents flowing through each conductor must be equal.

$$I_{upper} = I_{lower} = I \tag{5}$$

Combining this fact with equation 4 leaves us with

$$F_{mag} = \frac{\ell \mu_o I^2}{2\pi d} \tag{6}$$

The magnitude of the force exerted on the top wire by the bottom wire due to the current passing through each wire is directly proportional to that current squared if the separation distance between the wire centers, d, is held constant. This condition will be insured by utilization of the reflecting mirror connected to the upper bar and the laser beam reflected off of it. At equilibrium, with the bars separated by some distance d, the laser beam will reflect off of the mirror and strike the wall some distance b away from the mirror. Anytime the beam strikes the wall at this particular location, the conductors are separated by the particular distance d. This arrangement may also be used to determine the value of this distance d. If the conductors are forced to touch each other, the beam spot on the wall will lower by some distance D. If the horizontal distance from the knife-edge to the top conductor is measured to be a, simple geometry leads to this relationship to determine d_o, the distance between the conductor edges.

$$d_o = \frac{aD}{2b} \tag{7}$$

Now, d_o is the distance between the conductor edges, but we want d, the center to center distance between the conductors. The desired distance d is larger than d_o by an amount $2R$, where R is the radius of either conductor, assumed to be the same.

$$d = d_o + 2R \tag{8}$$

4 Procedure

1. Place the current balance assembly on a table opposite a wall or screen which is several meters away. Using the adjustable feet on the current balance, level the assembly. Make sure that the two conductors are as parallel as possible. A coin may be placed on the mass pan to hold the conductors together if adjustments of the lower conductor height are necessary. Adjust the counterpoise behind the mirror until the conductors are separated by an acceptable distance. A couple millimeter separation between the conductor edges is acceptable.

2. Position the laser such that it strikes the mirror and the beam is reflected onto the wall or screen. Note and mark the "equilibrium" position of the laser beam on the wall.

3. Connect the required circuit. The two conductors should be connected in series to the voltage supply. An ammeter arrangement must also be included in series so that the current may be accurately measured. Current must run in opposite directions through the wires. Make certain that the voltage is turned down to zero and then switch on the supply. As the voltage is raised slightly, the upper conductor should deflect upward and the laser beam spot on the wall should move. If this check does not work, something isn't set up properly.

4. Using the fractional mass set, place 10 mg of mass on the pan. This amount of mass should not cause the bars to touch. If it does, increase the equilibrium separation distance between the conductors slightly. When adding mass to the bar you must be careful not to disturb the placement of the bar or else the equilibrium point may be changed.

5. Adjust the current until the beam spot is back to the equilibrium position. Measure and record this current to the nearest $1/100^{th}$ of an amp. Note, if the current required for $10\,mg$ on the pan is larger than 3 or 3.25 amps, the conductors are too far apart. Adjust the counterweight and remeasure until the current is low enough for $10\,mg$.

6. Repeat steps 4 and 5, increasing the total amount of mass on the pan by $10\,mg$. Determine and record the amount of current required to return the laser spot to the equilibrium position.

7. Continue to repeat steps 4 and 5 as often as possible until the required current will exceed the maximum you are not to surpass. Your instructor will tell you this value. You should be able to at least acquire data for total masses up to $90\,mg$.

8. Turn off the voltage supply and place a coin on the mass pan such that the upper conductor rests on the lower conductor. Measure and record the distance the beam spot has moved from the equilibrium point, D.

9. Measure and record the following distances: ℓ – the length of the two conductors which carried current, a – the distance between the conducting bar and the support knife-edge, and b – the distance from the mirror to the screen at which the laser spot was observed. Also measure the diameter of the conductors.

5 Calculations

1. You will need to calculate the weight corresponding to the total mass you added to the top conductor for each trial.

2. Make a graph of Weight added vs current squared. Use a linear regression program to determine the slope and y-intercept of the best fit line to this data.

3. Use the slope and relevant measured lengths to determine an experimental value for μ_o. Repeat this calculation using the uncertainty in the slope of the best fit line which will result in the uncertainty in your experimental value for μ_o.

4. Check for agreement with the expected value of μ_o.

6 Questions

1. Starting with a picture, derive equation 7.

2. How could you modify the procedure such that you could repeat this experiment, resulting in data leading to the same graph, but have the current running through the conductors in the **SAME** direction?

3. An unknown current between 4and 6 amps is flowing through the conductors in opposite directions. Using the same setup you took data with, determine how accurately you would be able to determine the magnitude of this current. Justify with calculations and number.

Current Balance Data Sheet

Quantity	Length
a	
ℓ	
b	
diameter	
D	

Trial	Mass	Force	I	I^2
Units	mg	N	A	A^2
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
11				
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