

# RC Circuits

## 1 Object

To examine how the potential difference across a capacitor changes over time as the capacitor is charged or discharged through a resistor.

## 2 Apparatus

12-volt power supply, 220 – 1000  $k\Omega$  resistor, 100 – 470  $\mu F$  electrolytic capacitor, voltmeter, tap switch, stopwatch, alligator clips, lab wires.

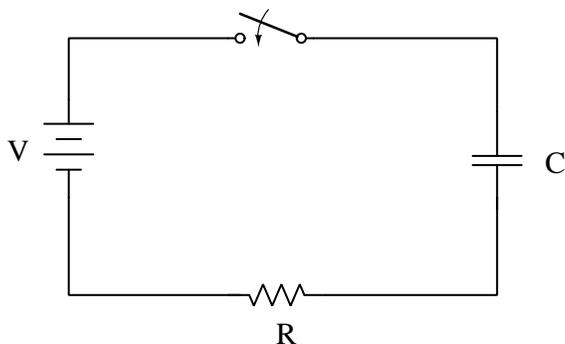
## 3 Theory

The amount of charge stored in a capacitor actually refers to the magnitude  $q$  of the charge that's been shifted from one of the capacitor plates to the other. The charge is directly proportional to the potential difference  $V$  between the plates, and the constant of proportionality is called the capacitance  $C$ :

$$q = CV \quad (1)$$

### 3.1 Charging a capacitor

When an uncharged capacitor is placed in an electrical circuit such that the plates are exposed to a potential difference, the capacitor does NOT charge instantly, as this would imply the existence of an infinite current over an infinitesimal time. Rather, the resistances in the circuit limit the current, and this affects how rapidly the capacitor can charge.



Consider the electrical circuit shown in the figure above. When the switch is closed, charges begin to flow into the upper plate and out of the lower plate of the capacitor. This flow of charges

constitutes a current  $i = \frac{dq}{dt}$  which flows in the wires of the circuit, but not across the gap of the capacitor. If we go around the circuit and apply Kirchoff's loop rule, we get the following:

$$V - \frac{q}{C} - iR = 0 \quad (2)$$

We can then use the expression from just above for the current and substitute into this equation. After a little algebra we can get the following form:

$$\frac{dq}{dt} + \frac{q}{RC} - \frac{V}{R} = 0 \quad (3)$$

This is called a differential equation, which can be solved in a straight forward way. The solution to this equation is a mathematical function for the charge on the capacitor as a function of time, and it looks like this:

$$q(t) = VC \left(1 - e^{-\frac{t}{RC}}\right) \quad (4)$$

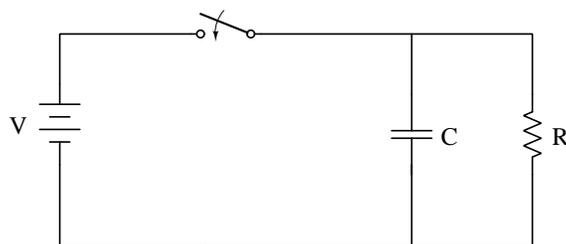
A more useful form is obtained by dividing both sides of this equation by the capacitance  $C$ , which gives us:

$$V_C(t) = V \left(1 - e^{-\frac{t}{RC}}\right) \quad (5)$$

where  $V_C$  is the voltage across the capacitor, and  $V$  is the power supply voltage. *This equation tells us how an initially uncharged capacitor's voltage changes as the capacitor charges.*

### 3.2 Discharging a capacitor

When an already-charged capacitor is discharged through a resistor, it does not discharge instantly. Consider the figure shown below. At some point in the past, the switch was closed and the capacitor became fully charged to a potential difference  $V$ . Once the switch is opened (the arrow on the switch going the opposite to that shown), the capacitor discharges directly through the resistor.



This should result in a situation where  $V_C$  decreases rapidly at first, but decreases more slowly over time. We can obtain the time-dependence of  $V_C$  for the case of discharging in a way similar to what we did for the case of charging. If we open the switch when  $t=0$ , then the solution to the differential equation above is:

$$q = VC \left(e^{-\frac{t}{RC}}\right) \quad (6)$$

If we again divide through by  $C$ , we get

$$V_C(t) = V e^{-\frac{t}{RC}} \quad (7)$$

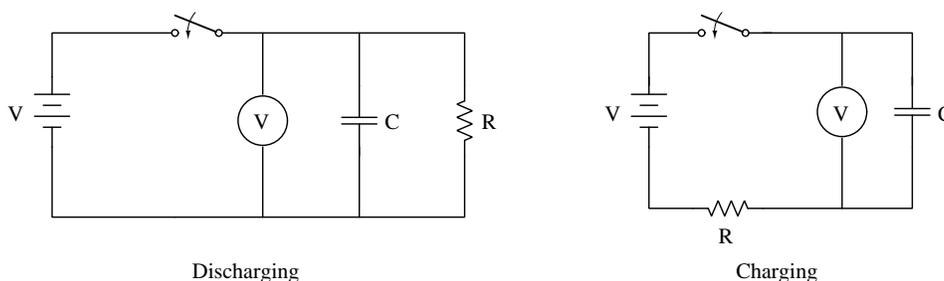
which tells us the voltage across the discharging capacitor as a function of time.

Notice that in both the charging and discharging cases, the behavior of the capacitor voltage over time depends on the product  $RC$ . This quantity must have units of time, and it's given the name *time constant*. Often the product  $RC$  is shortened to the Greek letter tau as

$$\tau = RC \quad (8)$$

In the case of charging, after one time constant the capacitor voltage has increased to a value that is  $1 - \frac{1}{e} = 63.2\%$  of its eventual maximum. In the case of discharging, after one time constant the capacitor voltage has decreased to a value that is  $\frac{1}{e} = 36.8\%$  of its initial value.

## 4 Procedure



### 4.1 Discharging a capacitor

1. Construct the circuit shown in the left figure above with the power supply off. Your instructor will tell you which capacitor and resistor to use. If the capacitor is an electrolytic type, then be sure to connect the + side to the positive potential side of the power supply.
2. Close the switch and adjust the power supply voltage until the voltmeter reads about 5.00 volts. Open the switch and watch what happens to the reading on the voltmeter.
3. Close the switch again and record the voltmeter reading as  $V$  when it stabilizes. Simultaneously open the switch and start a stopwatch. Record the voltmeter reading every 15 seconds for a total time of 5 minutes.
4. Remove the resistor from the circuit and measure its resistance using an ohmmeter. Record this as your  $R$  value. The uncertainty in this measurement is insignificant compared to the uncertainties in the other measurements, so we will ignore it.

### 4.2 Charging a capacitor

1. Take apart the circuit from Part 1. Connect a voltmeter directly across the power supply and set the supply voltage to about 5.00 volts, recording this value as  $V$ . Then remove the voltmeter and DO NOT change the voltage setting afterwards.

2. Use the same resistor and capacitor from Part 1 and construct the circuit shown in the right figure above. Briefly connect an extra wire across the terminals of the capacitor to make sure that it is discharged. If you've done this correctly, your voltmeter should read zero volts.
3. Simultaneously close the switch and start a stopwatch. Record the voltmeter reading every 15 seconds for a total time of 5 minutes.

## 5 Calculations

### 5.1 Discharging a capacitor

1. Make a plot of the voltage across the capacitor as a function of time, and be sure to include the  $t = 0$  value. Does it look like it should? Add a title and axis labels and print it out. Extract the value of the time constant  $\tau$  by hand by examining your graph and estimating the time at which the potential has dropped to  $1/e = 36.8\%$  of its initial value. Draw a vertical line at this estimated time. Make a reasonable estimate of the uncertainty in your value for the time constant  $\tau$  as well by drawing error bars on the time axis to indicate the range. Report your estimate of the time constant  $\tau$  and its associated uncertainty in a 2-line summary in your spreadsheet, but not on the graph itself.
2. If we take Eq. 7 above and compute the natural logarithm of both sides, we end up with a formula for a straight line:

$$\ln V_C = \ln V - \frac{t}{RC} \quad (9)$$

Compute the natural logarithm of the voltage across the capacitor for each of your measurements during discharging. Then make a plot of  $\ln(V_C)$  versus time, and perform a linear regression. The values of  $\ln(V_C)$  or  $\ln(V)$  have no units for this plot. Record your regression results on the graph and also on the spreadsheet. You DO have a theoretical value for  $\ln(V)$  (4-line summary!). Although you made a direct ohmmeter measurement of the resistance  $R$ , you should NOT trust the manufacturer's capacitance value if it is written on the side of the capacitor. This means you do NOT have a theoretical value for  $C$ , and consequently do NOT have a theoretical value for the slope (2-line summary!).

3. Use Eq. 8 in the handout and the slope you got from the regression to compute the value of the time constant  $\tau$ . Estimate the the uncertainty as  $\delta\tau = \tau \cdot \frac{(\text{uncertainty in slope})}{|\text{slope}|}$ . Report this result in a 2-line summary on the spreadsheet.
4. Use Eq. 8 in the handout, the time constant  $\tau$  you just obtained from the regression, and the measured resistance  $R$  to compute a value for the capacitance  $C$ . Estimate your uncertainty in the capacitance as  $\delta C = C * \frac{\delta\tau}{\tau}$ . Report this result in a 2-line summary on the spreadsheet.
5. Use the y-intercept to estimate the value of the initial capacitor voltage. Report this voltage value. Compare this voltage value to your actual measured value of  $V$  and calculate the percentage difference. Report this percent difference.

## 5.2 Charging a capacitor

1. Make a plot of the voltage across the capacitor as a function of time. Adjust the vertical axis range so that it goes from 0.0 volts to something larger than  $V$ . Does your plot look like it should? Add a title and axis labels and print it out. Draw a horizontal line on the graph at the maximum voltage. Extract the value of the time constant  $\tau$  by hand by examining your graph and estimating the time at which the potential has risen to  $1 - (1/e) = 63.2\%$  of  $V$ . Draw a vertical line at this estimated time. Make a reasonable estimate of the uncertainty in your value for the time constant  $\tau$  as well by drawing horizontal error bars on the time axis to indicate the range. Report the values of your estimate of the time constant  $\tau$  and its associated uncertainty in a 2-line summary in your spreadsheet, but not on the graph itself.
2. If we take Eq. 5 in the handout and rewrite it as

$$1 - \frac{V_C}{V} = e^{-\frac{t}{RC}} \quad (10)$$

and then compute the natural logarithm of both sides, we end up with a formula for a straight line:

$$\ln\left(1 - \frac{V_C}{V}\right) = -\frac{t}{RC} \quad (11)$$

3. Divide each of the capacitor voltage values you measured over time by the value of  $V$ . Next, compute the values of the natural logarithm of  $1 - (V_C/V)$  at these times. Make a plot of these logarithm values versus time and perform a linear regression. Record your regression results on the graph and also on the spreadsheet. You DO have a theoretical value for the y-intercept (4-line summary!), but NOT for the slope (2-line summary!).
4. Use Eq. 8 in the handout and the slope you got from this regression to compute the value of the time constant  $\tau$ . Estimate the uncertainty as  $\delta\tau = \tau \cdot \frac{(\text{uncertainty in slope})}{|\text{slope}|}$ . Report this result in a 4-line summary, using the value of  $\tau$  you obtained from the first regression as a theoretical value.
5. Use Eq. 8 in the handout, the time constant  $\tau$  you just obtained from the regression, and the measured resistance  $R$  to compute a value for the capacitance  $C$ . Estimate your uncertainty in the capacitance as  $\delta C = C * \frac{\delta\tau}{\tau}$ . Report this result in a 2-line summary on the spreadsheet.
6. Compare the two values of capacitance you obtained in the discharging and charging situations. Find the percentage difference and report it.
7. Average your two capacitance values and report this average. Read the manufacturer's value for the capacitance directly from the outside of the capacitor and compute the percentage difference of this from your average value for  $C$ . Report this percentage difference.

## 6 Questions

1. A 50.0 microFarad capacitor is connected to a battery having an internal resistance of 3.20 Ohms. How long will it take for the voltage across the capacitor to reach 99.0% of its final

value? Give your answer to 3 sig. figs.

2. When a capacitor discharges through a resistor, how long does it take for the capacitor voltage to reach half of its initial value? Give your answer as a multiple of the time constant  $\tau$ .
3. You need an RC combination for a digital timer that will allow a capacitor to charge to 75% of its maximum voltage in 3.00 milliseconds. Resistors and capacitors of arbitrary value are not available, but only come in certain standard values. Go online and find a website that sells resistors and capacitors and find a resistor and capacitor combination which will give a time constant within 1% of what is needed. Report the value of  $R$  and  $C$  which you've found and where you found it.