Ideal Gas Law

1 Object

To demonstrate the dependence of pressure on temperature for a fixed volume of real gas.

2 Apparatus

Constant volume gas apparatus with pressure gauge, fixed mass/volume of air, and four liquid baths at various fixed temperatures, digital thermometer, hairdryer.

3 Theory

In this lab you will be exploring how a gas at fixed volume and density behaves as the temperature varies. For a dilute gas in equilibrium, far from the liquid/vapor boundary, the ideal gas law should be valid. The ideal gas law may be stated in equation form as:

\[ PV = N k_B T \]  

Here, \( P \) is the (absolute) pressure, \( V \) is the volume, \( N \) is the number of gas molecules, \( k_B \) is the Boltzmann Constant and \( T \) is the absolute temperature.

In the case of an ideal gas, the pressure arises from the collisions between the molecules and the surfaces of any object in contact with the gas. In the case where the molecules are enclosed in a rigid container, \( V \) and \( N \) are constant. We can solve for \( P \) by dividing both sides of equation 1 by \( V \) and then we have \( P \) as a linear function of \( T \) as follows:

\[ P = \frac{N k_B T}{V} \]  

Here it becomes clear that the temperature must be expressed in terms of an *absolute scale*, such as the Kelvin scale, such that \( T \) goes to zero at absolute zero. If we express \( T \) in °C, we would expect the pressure of our gas to go to zero when it reaches the freezing point of water. That makes no sense, because we would then have no air to breathe during winter! If we wish to work in degrees Celsius, we can call the pressure when the temperature is 0 degrees \( P_0 \), and then equation (2) becomes

\[ P = \frac{N k_B T}{V} + P_0 \]  

Note that this equation is in the form of a line, \( y = mx+b \), where the slope \( m = \frac{N k_B}{V} \).
Since the pressure of an ideal gas can be understood as originating from the collision of the
gas molecules with surfaces, we can understand absolute zero as the temperature at which the
molecular motion ceases. It is the point at which the kinetic energy of all the gas molecules is
zero. Since kinetic energy can only be positive or zero, it makes sense that temperature on an
absolute scale can also only be either positive or zero.

In S.I. units, our temperature is expressed in kelvin, our pressure in pascals (Pa), and our volume
in m$^3$. Fortunately, a change of 1 K is equal to a change of 1 $^\circ$C, so we also can say that the Boltzmann constant $k_B = 1.38 \times 10^{-23}$ J/$^\circ$C. With this fact, we can then find absolute zero on the Celsius scale by finding where the pressure actually extrapolates to 0 in equation 3, which will be part of the calculations for this lab.

What about the value of our slope from equation (3), $m = Nk_B/V$? This value depends on $N/V$, the number of gas molecules ($N$) that are enclosed in a volume ($V$), so $N/V$ is the number density of your gas. That means your slope $m = Nk_B/V$ is the number density of your gas multiplied by the Boltzmann constant.

### 4 Procedure

**YOU MUST BE WEARING EYE PROTECTION AT ALL TIMES DURING THIS LAB.**

This lab involves temperature extremes that have the potential for severe burns including boiling water, dry ice in alcohol and liquid nitrogen. Burns from extreme cold are just as harmful as burns from heat. Avoid touching the metal bulb with your bare skin at any time as a precaution.

![Figure 1: Constant Volume Gas Apparatus](image-url)
1. Each group should select a constant volume gas apparatus. Hold it upright with the center of the gauge at nose height, and note the pressure using the inner scale on the gauge, which is in kilopascals (kPa). **Gently** tap the pressure gauge and note the pressure again. When reading the pressure, it is important to hold the gauge in the same position relative to your eyes and read it the same way each time for consistency. Reading the gauge from inconsistent angles will introduce random error. If your gauge’s scale runs from -100 kPa to 100 kPa, it tells you the relative pressure compared to the air pressure in the room. If you have one of these relative gauges, you should add 94 kPa to every one of your gauge readings to get \( P \). 94 kPa is the average atmospheric pressure at 600 m above sea level, which is the approximate elevation of the Gonzaga campus.

2. In the data table on page 5, write down the letter or number scratched into the back of the metal just below the pressure gauge.

3. Use your thermometer to note the temperature of the room. Compare your initial temperature and pressure results with the values written on the label on the back of the pressure gauge and make sure they are similar.

4. There are 4 different types of temperature baths at the stations around the room. Your instructor will tell you how to rotate between stations.

   (a) Boiling water bath- Turn the hot plate to “1000” (which indicates 1000 W) if it is not already on. The bath should be at a rolling boil. You may check the bath temperature with a thermometer. The boiling water bath temperature should be close to 98.0 °C.

   (b) Ice water bath- The bath should be a mixture of about 50% liquid tap water and 50% ice. The ice water bath temperature is 0 °C.

   (c) Dry ice alcohol bath- This bath consists of frozen carbon dioxide (dry ice) mixed with liquid 2-propanol. **This bath is cold enough to cause severe frost burns. Do not put your hands inside the container and do not attempt to steady the container with your hands during immersion.** Do not put any thermometers in this bath. The temperature is -72 °C. You should see several chunks of dry ice floating in the bath. Let your instructor know if the dry ice appears to be running low.

   (d) Liquid nitrogen bath- Your instructor will handle this station. The temperature of the liquid nitrogen bath is -197 °C. **Do not attempt to use the liquid nitrogen station.**

5. Ensure that the metal bulb at the bottom of the gas apparatus is **completely dry by wiping it with a cloth**. Then, if you are at any of the stations (a) through (c) immerse the metal bulb fully in your bath. You may gently and slowly move the bulb around to mix the bath while you wait for the pressure gauge to stabilize. Gently tap the gauge and note the pressure and the type of bath. At station (d), give your apparatus to your instructor once your metal bulb is dry. Again, **do not attempt to use the liquid nitrogen station yourself.**
6. Remove the bulb from the bath and gently place the bulb on the towel next to the station and lay down the apparatus. **Do not touch the bulb with your bare hands at any time, even if you believe it to be near room temperature.** Wait a few minutes for the bulb to return near room temperature. If the bulb has frost on it, you may use a hair dryer to warm the bulb until it no longer frosts. Wipe the bulb with a cloth after it is near room temperature. For stations (a) and (b), you may choose to do a second immersion and record the final pressure a second time. Do not do repeat immersions at stations (c) or (d).

7. Rotate through the stations, repeating steps 4 & 5. Again, at the liquid nitrogen station, when told, hand your apparatus to your instructor.

8. Exchange your gas apparatus for one with a different room temperature pressure, and thus different gas density. Repeat all steps above with the second apparatus.

5  **Calculations**

1. Plot Pressure in Pa versus Temperature in °C. Perform a linear regression to determine the value of \( Nk_B/V \) for each of your two data sets. Recall that the units of pressure on the gauge are kPa, not Pa. Determine the value of \( N/V \), the number density of gas molecules.

2. Calculate the theoretical values of \( N/V \) for each of your two data sets. To do this plug your value of \( P_0 \) into the equation:

   \[
   \frac{N}{V} = \frac{P_0}{(273°C \times k_B)}
   \]

   where \( P_0 \) is the pressure you read at station (b), the ice bath.

3. For each of your two data sets, use a 4-line summary to compare the value of \( N/V \) from step 1 above to the value from step 2.

4. Plot Temperature versus Pressure now. Perform a linear regression analysis on each data set and use a 4-line summary to compare your y-intercept, where pressure is 0, with the value of absolute zero on the Celsius scale, which you may look up online or request from your instructor.

6  **Questions**

1. Why is the temperature of the boiling water bath not 100 °C? Is it related to the reason that the outdoor air pressure in Spokane is always less than 1 atmosphere = 101.3 kPa?

2. How do we know the gas density does not change within the gas apparatus even when the temperature varies greatly?

3. What happens when humid air enters a cool or cold environment? If the air in a gas apparatus was very humid at room temperature, how might this affect your results?
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