WORK - ENERGY

Object

To investigate the work-kinetic energy relationship and conservation of energy.

Apparatus

Track and associated stops, one dynamics cart, one photogate timer, interface equipment, masses, calipers, meterstick and a mass balance.

Theory

The work kinetic energy relationship gives us a new way, other than forces and kinematics, to understand physical systems. One does work by exerting a force, on a moving object, along the line of that object's motion. Work is an attempt to change the speed of an object. If a force is exerted at some angle with respect to the direction of motion, only the component of that force along the line of motion does work. If a constant force $\mathbf{F}$ acts on an object as it undergoes some displacement $\mathbf{s}$ along a straight line such that θ is the angle between the direction of $\mathbf{F}$ and the direction of $\mathbf{s}$ then the work done by that force on the object is given by

$$\text{Work} \;=\; F \; s \; \cos{(\theta)} \;=\; F_{||} \; s \tag{1.1}$$

If the parallel component of the force F is not remaining constant, then calculus, or graphical techniques, must be used to determine the work done by a force. If the work done by each individual force acting on an object as it undergoes some displacement, can be found, these individual works may be added together, as scalers, to determine the net work done on an object. Rather, one could first add the forces together, as vectors, to determine the net force acting on an object during some displacement and the work done by this net force would also equal the net work done on an object.

$$Work_{net} \;=\; \sum_{i} \text{Work}_{i} \;=\; \text{F}_{\text{net}} \; s \; \cos{(\theta)} \tag{1.2}$$

The net work done on an object, during any motion, is equal to the change in the object's kinetic energy during that motion. Kinetic energy is a measure of an object's ability to do work, stored in the motion of that object. Kinetic energy depends on the mass of the object and the speed with which that object is moving.

$$Work_{net} \;=\; KE_{Final} \;-\; KE_{Initial} \;=\; \frac{mv_f^2}{2} \;-\; \frac{mv_i^2}{2} \tag{1.3}$$

This equation is the work-kinetic energy relationship. The work kinetic energy relationship is sometimes written in a more convenient form for situations involving "conservative" forces. A conservative force is one for which the work done on an object as it moves from one point to another is independent of the path taken between those points, rather the work only depends on the initial and final positions. In such a case, the work done by that conservative force can be expressed in terms of the negative of the change in a potential energy function associated with that conservative force. Each conservative force has its own potential energy function.

$$Work_{cons} \;=\; -\;(PE_{Final} \;-\; PE_{Initial}) \tag{1.4}$$

The two most common conservative forces that come up in first semester physics classes are the gravitational force exerted on a mass by the earth and the force exerted by a spring. The potential energy functions associated with those forces are

$$PE_{grav} \;=\; m \; g \; y \qquad PE_{spring} \;=\; \frac{k \; stretch^2}{2} \tag{1.5}$$

where y is the vertical location of the mass m and k is the spring constant of the spring. If the net work in equation 1.3 is split into two parts, work done by conservative forces and work done by nonconservative forces, the work done by conservative forces can all be accounted for using the changes in the appropriate potential energy functions.

$$Work_{net} \; = \; Work_{nc} \; + \; Work_{cons} \; = \; Work_{nc} \; - \; (PE_{Final} \; - \; PE_{Initial}) \tag{1.6}$$

The PE terms are understood to refer to the sum of all potential energy functions in the system, one for each conservative force that does work. Combining this relationship with the work kinetic energy relationship and regrouping terms one gets

$$Work_{nc} \; = \; (KE_{Final} \; + \; PE_{Final}) \; - \; (KE_{Initial} \; + \; PE_{Initial}) \; = \; E_{Final} \; - \; E_{Initial} \tag{1.7}$$

where we have defined E, the mechanical energy, to be the sum of the potential and kinetic energy of a system. If there is no work done by nonconservative forces, it follows from equation 1.7 that the initial mechanical energy must equal the final mechanical energy; i.e. the mechanical energy is conserved if there is no work done by nonconservative forces.

$$KE_{Initial} \; + \; PE_{Initial} \; = \; KE_{Final} \; + \; PE_{Final} \tag{1.8}$$

Let's see how these relationships can be applied to the physical systems you will be studying during this lab.

Part 1. Cart on an inclined track.

A cart is released from rest on a track inclined at some angle θ with respect to the horizontal. We'll assume friction and air resistance are negligible. As the cart moves down the incline, only two forces act on it; gravity and the normal force. Since the normal force acts perpendicular to the motion of the cart, it cannot do any work on the cart. Only the parallel component of the gravitational force, mgsin(θ), does work on the cart. If s is the distance the cart moves down the incline then equation 1.3 becomes

$$Work_{net} \; = \; Work_{grav} \; = \; m \, g \, \sin(\theta) \, s \; = \; \frac{mv_f^2}{2} \tag{1.9}$$

Part 2. Cart on an inclined track with string connected to mass.

In this case, the cart will be on the incline connected, via a light string over a pulley at the upper end of the track, to a mass which may fall towards the ground. As the mass falls, the cart will be pulled up the incline the same distance s that the cart falls. The pulley may be assumed to be massless and frictionless so that the tension everywhere in the string is assumed to be uniform. Any frictional force on the cart from the incline is assumed to be negligible as is air resistance. In that case, the only non conservative force that may do work is the tension in the string. However, the tension would do just as much positive work on the cart as it does negative work on the hanging mass. As a result, the total work done by non-conservative forces will be zero. The mechanical energy of the system should be conserved as the objects move. If the system is released from rest, the initial kinetic energy will be zero. As the system begins to move, the mass will move downward and lose gravitational potential energy. At the same time, the cart will be moving upward thus gaining gravitational potential energy. If the mass loses more gravitational energy than the cart gains, the difference will be converted into kinetic energy in the system. Both the mass and the cart will have to move with the same speed. Our conservation of energy relationship can then be applied to this system.

Procedure

Part 1. Cart on an incline.

1. Measure and record the mass of your cart.
2. Set the track to an angle of inclination of around 8 degrees to the horizontal. Determine θ accurately by measuring distance and using trig.
3. Set the photogate up near the bottom of the track such that the small tube attached to the cart will block the beam but nothing will collide with the photogate. Determine the location of the cart when the tube is centered in the photogate beam. This is the "final" position of the cart during its motion on the track.
4. Place the cart 1.00 meters from the "final" position on the track. Release the cart from rest letting it move down the ramp and through the photogate. Record the time it blocks the photogate beam. Be sure and catch the cart after it passes the photogate so it does not crash off of the track.
5. Repeat twice more for a total of 3 measured times for this distance d. Record the distance the cart moved from release until it reached the final position and the 3 measured times.
6. Repeat at 7 other distances along the track spanning distances from 15 cm up to the maximum possible distance.
7. Using the drill bits, as in the ballistics lab, collect data and determine the width of your photogate beam. Use calipers to measure the diameter of the small pvc tube attached to your cart which blocked the photogate beam. Record this data.

Part 2. Cart on incline with pulley and hanging mass.

1. Attach a pulley to the upper end of the track. Attach a length of string to the cart such that when the cart is at the lower end of the track the string just extends over the pulley far enough to connect to a mass hanger. Move the mass hanger to the ground thus pulling the cart up the track. Place the photogate such that it would have finished being blocked by the cart just before the mass hits the ground. Determine the position of the cart when it is centered while blocking the photogate beam. This is the "final" location of the cart. Move the cart back down the incline until the mass hanger is back as close to the pulley as possible. This is the "initial" location of the cart. Record the distance, s, between these two locations.
2. Attach enough mass to the hanger such that it will cause the cart to move up the track when released from rest. Place the cart at the initial position and release it from rest. Record the amount of time the cart blocked the beam. Repeat for two more measurements giving three measured times. Record the total amount of mass m that was attached to the string over the pulley.
3. Repeat for 7 other attached masses, spanning a range of several hundred grams, always releasing the cart from rest at the same location. Record three times for each mass.

Calculations

Part 1. Cart on an incline.

1. Determine the distance that your cart moved while it was blocking the photogate. This distance will be the same throughout the rest of your calculations.
2. For each of your eight distances, calculate the average of your three measured times. Use this time and the distance the pvc tube moved while blocking the photogate beam to determine the speed of the cart at the "final" position.
3. Calculate the kinetic energy of the cart at the "final" position. Since the initial kinetic energy of the cart was 0 Joules, the "final" kinetic energy is the change in kinetic energy.
4. Calculate the work done on the cart by gravity. Do this calculation using your knowledge of the distance the cart moved, the magnitude of the gravitational force on the cart and the angle of the ramp; not using the work kinetic energy relationship. Equation 1.1 will be useful here. If we neglect any frictional effects, the work done

by gravity is equal to the net work done on the cart as it moves down the incline.

5. Construct a graph of work done by gravity vs the change in kinetic energy. Perform a linear regression and compare the slope and y-intercept of your best fit line with expected values. You may find equation 1.9 useful in determining the expected values for the slope and y-intercept.

Part 2. Cart on incline with pulley and hanging mass.

1. Determine the average time to block the photogate for each of your eight different attached masses. Also calculate the speed of the cart and the hanging mass at their "final" positions. Use this speed to find the total kinetic energy of the system when the cart and the hanging mass reach the "final" position.

2. Calculate the initial and final potential energy for the cart as it moved up the incline from the "initial" to the "final" position. This value should be the same for all eight attached masses as it does not depend on the attached mass. Clearly indicate where the vertical origin (where you set $y = 0$) for your cart is located; a note in the lab report would be sufficient.

3. Calculate the initial and final gravitational potential energy for the hanging mass while the cart moved up the incline from the "initial" to the "final" position for each of the eight different attached masses. Again, clearly indicate where the vertical origin for your attached mass is located.

4. Calculate the initial and final mechanical energy for the system for each of the eight setups.

5. Construct a graph of final mechanical energy of the system vs initial mechanical energy of the system. Perform a linear regression and compare the slope and y-intercept with expected values.

Questions

1. In part 2 of the lab, is the mechanical energy of ONLY the cart conserved as it moves along the ramp? Explain.

2. For part 1 of the lab, assuming all measurements and calculations were done correctly, what would a slope significantly less than 1 tell you about the system? What would a slope significantly greater than 1 tell you about the system? Explain.

3. In part 2, if you decided to make the origin for measuring your cart's vertical position 0.5 meters lower than you did, how would that change your calculated value for the initial mechanical energy of the system? Come up with a numerical value! What would the change in your calculated value for the final mechanical energy be? Explain how you would expect the slope and y-intercept of your graph for part 2 to be affected.

WORK-ENERGY DATA SHEET

Part 1 - Cart on an incline

Cart Mass	
Ramp Angle	

Trial	Distance	Time 1	Time 2	Time 3
Units				
1	1.00			
2				
3				
4				
5				
6				
7				
8				

Part 2 - Cart on incline with pulley and hanging mass

Distance (s)	

Trial	Attached Mass	Time 1	Time 2	Time 3
Units				
1				
2				
3				
4				
5				
6				
7				
8				