Rotational Dynamics

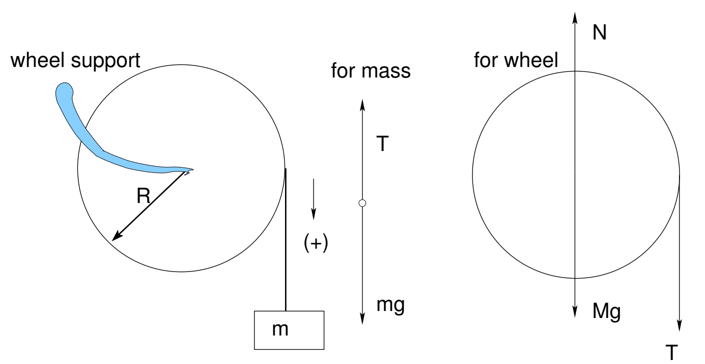
1 Object

To experimentally determine the moment of inertia of a bicycle wheel about its center as well as the frictional torque in the wheel's bearings, by applying the ideas of torque, angular acceleration, and translational motion.

2 Apparatus

Bicycle wheel with small pulley mounted on it, string, weights, stopwatch, caliper, meter stick, and graphing program.

3 Theory



Consider a bicycle wheel which is mounted such that it may rotate about its center, as it does when a bike is ridden. Assume a string is wrapped around the wheel in some manner, either about its outer rim or on a small pulley concentric with the axle. The free end of this string is connected some mass hanger with mass attached. If released, the mass will accelerate toward the ground. One may sketch a free body diagram for the total mass m hanging from the string and apply Newton's 2^{nd} law to find that

$$ma = mg - T \qquad or \qquad T = m(g - a) \tag{1}$$

where m is the total mass hanging from the string, $g = 9.8m/s^2$ is the acceleration due to gravity, and T is the tension in the string. If we knew the acceleration of the hanging mass, we could determine the tension in the string. We know that the tension in the string should be constant, assuming all other forces remain constant with time, which means that the acceleration of that hanging mass should be constant. Since the mass starts from rest, if we knew the final and initial vertical locations of the mass and the time it took to fall, we could calculate the mass's acceleration using

$$y = y_o + \frac{at^2}{2} \qquad or \qquad a = \frac{2(y - y_o)}{t^2} \tag{2}$$

where it may be convenient to choose the initial location of the mass to be $y = 0$ and down to be the positive direction.

Now, if the mass starts from rest and falls down, accelerating at some rate a, it must move faster and faster. As a result, the wheel must spin faster and faster meaning that it must have a non-zero angular acceleration. There is a relationship between the distance the mass falls and the angle through which the wheel must rotate,

$$\theta = \frac{(y - y_o)}{r} \tag{3}$$

where θ is given in radians and r is the radius about which the string is wrapped. This equation simply means that the wheel will have turned through one full revolution when the mass has fallen a distance of $2\pi r$. Taking time derivatives of equation 3 yields relationships between the other angular kinematic quantities associated with the wheel and the translational kinematic quantities associated with the falling mass

$$\omega = \frac{v}{r} \qquad and \qquad \alpha = \frac{a}{r} \tag{4}$$

where v is the speed of the mass and a is the acceleration of the mass. We see that if the mass is to accelerate, the wheel must accelerate angularly. The only way the wheel can accelerate angularly is if there is a nonzero net torque exerted on the wheel. Since the tension force is acting on the wheel both a distance r from the axis about which it rotates and perpendicular to $\vec{r}$, the tension force causes a torque on the wheel in the direction in which it accelerates angularly. Any friction in the bearings would cause a torque in the opposite direction, trying to slow the wheel down. As a result, the rotational equivalent of Newton's second law may be written as

$$\vec{\tau}_{net} = I_{wheel}\vec{\alpha} \qquad or \qquad \tau_T - \tau_{fric} = I_{wheel}\alpha \tag{5}$$

where the moment of inertia of the wheel is about the axle – the wheel's axis of rotation. The tension in the string will be the one which results in an acceleration of the hanging mass and an angular acceleration of the wheel which satisfies equation 4. The torque due to the tension in the string would depend on both r and T such that

$$\tau_T = rT\sin(90) = rT \tag{6}$$

since $\sin(90) = 1$.

Could you make a theoretical calculation of what the moment of inertia for the wheel about its axle should be? Well, the majority of the mass of that wheel is in the rim out at a radius R. Then there are a bunch of spokes which look like rods of length R rotated about their ends. If we assume there are N spokes, all of the same mass, then we could make a theoretical calculation of the bicycle wheel's moment of inertia as follows,

$$I_{wheel} = m_{rim}R^2 + N\frac{m_{spoke}R^2}{3} \tag{7}$$

where the hub is being ignored.

4 Procedure

4.1 String on small pulley

1. The bicycle wheel should be mounted such that the wheel is free to rotate. There is a string wrapped around the outer rim of the wheel, the end of which should be temporarily taped to the rim. There should also be a string wrapped around the small pulley which is also temporarily taped to the rim. The mass of the rim should be marked somewhere on the side of the rim. Record this value on the data sheet. There should also be some spare spokes in the lab room. Grab one of the appropriate length, determine and record its mass. Count the number of spokes on your wheel and record that number as well.

2. Untape the end of the string wrapped around the small pulley. attach a mass hanger to this end of the string. If you release the mass hanger, the hanger may start to accelerate to the floor as the string unwinds and the wheel starts to turn. You don't want this to happen yet. Adjust the location of the mass hanger until it is at a location which you can precisely reproduce. Measure and record the distance from the bottom of the hanger to the floor. This distance should be larger than $1.5\,m$.

3. Release the mass, from rest, and it will fall to the ground. Measure the time it takes to reach the ground using a stopwatch. Record the total amount of mass on the string and the time taken to fall. Repeat the process, with the same total mass, 3 more times recording each time taken to fall to the ground.

4. Repeat for 7 other values of total mass hanging on the string. These 8 values should span a range from $50\,g$ up to $250\,g$.

5. Measure and record the radius of the small pulley. Using a caliper will probably yield the most accurate measurement.

4.2 String on outer rim.

1. Remove the mass hanger and rewind the string on the small pulley. Use a small piece of tape to affix the string to a spoke so that it will not unwind as the wheel turns. Untape the end of the string which is wrapped around the outer rim. Instead of using the 50 gram mass hanger, attach a paper clip to this string such that you can hang masses on the paper clip. You may assume the mass of the paper clip is negligible.

2. Hang a 20 gram mass on the paper clip and the wheel should start to rotate. Again, stop the wheel and adjust it such that the bottom of the mass is in a reproducible location which is more than $1.50\,m$ above the ground. Measure and record this distance.

3. Release the mass, measure and record the time it takes to fall to the ground. Repeat this measurement three more times for this particular mass.

4. Repeat this process for seven other values of total added mass. These values should span a range from 20 grams up to 120 grams.

5. After completing these procedures, remove the extra masses and tape the strings to the wheel so they will not unwind.

6. Mark a location on the rim with a piece of tape that you can easily see as the wheel rotates.

7. Give the wheel a bit of a spin. When the mark reaches the very top, start a stopwatch running and count the number of revolutions the wheel makes until it comes to a stop. Stop the stopwatch when the wheel stops spinning in the direction it was going. Record the number of revolutions, to at least the nearest $\frac{1}{8}^{th}$ of a revolution, and the time it took to stop. The wheel should make at least 10 complete revolutions. If not, spin it faster and try again.

5 Calculations

5.1 Small Pulley

1. For each of the values of total added mass, calculate the average time taken to fall. Also determine the associated error with this mean value. These values should be listed in a table in the lab write up.

2. Use this mean time taken to fall to calculate the acceleration of the mass as it fell toward the ground using equation 2. These values should be listed in the above-mentioned table.

3. Knowing the acceleration, calculate the tension in the string as the mass accelerated toward the ground. These values should also be listed in the table.

4. Calculate the angular acceleration of the wheel using the determined translational acceleration of the hanging mass. List these values in the table.

5. Calculate the torque exerted on the wheel by the tension in the string as the hanging mass fell to the ground. Also list these values in the table.

6. Construct a graph of torque exerted on the wheel vs resulting angular acceleration of the wheel. Determine the slope and intercept of the best fit line to this graph. From these values determine experimental values, and associated uncertainties, for the moment of inertia of the wheel and the torque on the wheel due to friction in the bearings.

5.2 Outer Rim

1. Repeat the above calculations for the data involving the string wrapped around the outer rim of the wheel. List these results in a separate table.

2. Construct a graph using these values of torque and angular acceleration and again determine an experimental value for the moment of inertia of the wheel and the torque on the wheel due to friction in the bearings. Check to see if these results are consistent with those of the first graph.

3. Calculate a theoretical value for the moment of inertia of the wheel using the recorded values and equation 7. How does this result compare with the results from the two graphs? Discuss the comparisons in your error analysis.

4. Using the data from spinning the wheel and timing how long it takes to stop and the rotational kinematics equations in you text, determine the average angular acceleration of the wheel while it came to a stop.

5. This angular acceleration is the result of the torque due to friction in the wheel bearings, air resistance, ... Calculate this torque by multiplying the determined angular acceleration by the theoretical moment of inertia of the bicycle wheel. We will call this a theoretical value for the torque due to friction. Check to see if this value is in agreement with your results from each of the graphs.

6 Questions

1. For which rim (inner or outer) would one see a greater acceleration upon hanging 100 grams and releasing the system? Explain the physics behind this result.

2. How negligible is the moment of inertia of the hub compared to the moment of inertia for the whole wheel? Do a rough calculation to compare these values. Show your work and explain your results.

3. Do you expect the y intercept of your graphs to be zero? If the y intercept is positive, what would that mean? If the y intercept is negative, what would that mean? Discuss and explain your particular results in your error analysis.

Rotational Dynamics Data Sheet

Mass of Rim	Mass of Spoke	Number of Spokes

Radius of Pulley		Radius of Rim	
Distance Part 1		Distance Part 2	

Part 1 - String wrapped around small pulley

Total Mass	Time 1	Time 2	Time 3	Time 4	Mean Time	Assoc Error
grams	sec.	sec.	sec.	sec.	sec.	sec.

Part 2 - String wrapped around outer rim

Total Mass	Time 1	Time 2	Time 3	Time 4	Mean Time	Assoc Error
grams	sec.	sec.	sec.	sec.	sec.	sec.

Friction data

number of rotations: ____________________ time (s): ____________________