

Linear Momentum

1 Object

To investigate the conservation of linear momentum in a system of colliding objects moving in one dimension. Also to see how the momentum of a system changes when a net impulse is imparted to it.

2 Apparatus

Track and associated stops, two dynamics carts, mass block, two photo-gate timers, one force sensor, interface equipment, calipers and mass balance.

3 Theory

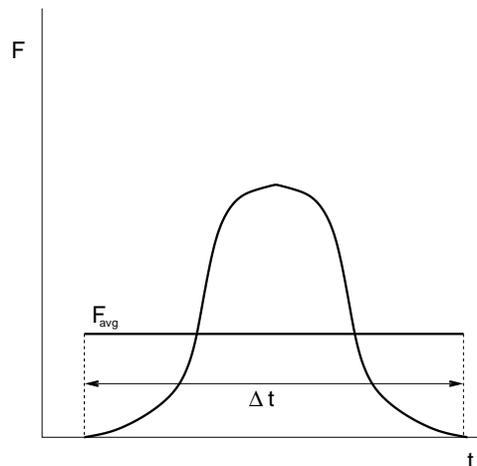
This experiment involves objects colliding while undergoing one dimensional motion. We will be looking at the relationships between linear momentum and the impulse imparted to a system of objects. Any mass m moving with some nonzero velocity, \vec{v} , has a vector *momentum*

$$\vec{p} = m\vec{v} \quad (1)$$

Impulse is imparted to an object or system of objects any time a force is exerted on that object or system of objects for some interval of time.

$$\text{Impulse} = \vec{I} = \int \vec{F} dt = \vec{F}_{avg} \Delta t \quad (2)$$

Graphically, $\int F dt$ is the area beneath the curve of an F versus t plot. Usually, the force varies as a function of time, so the curve has a shape, for example, as shown in the following figure. Again,



the integral is just the area bounded by the force curve and the horizontal axis during the specified time interval. Area above the time axis is positive and area below the time axis is negative. What is meant by the average force (F_{avg}), then, is the constant force which could be applied over the same time interval such that the bounded area would be the same as for the time varying force. If more

than one force acts on a system then the net impulse imparted to that system is the sum of the impulses imparted by each individual force. This net impulse is equal to the change in momentum of that system during the relevant time interval.

$$\vec{I}_{net} = \sum \int \vec{F} dt = \int \sum \vec{F} dt = \int \vec{F}_{net} dt \quad (3)$$

using calculus and knowing force as a function of time. We could equivalently write

$$\vec{I}_{net} = \sum \vec{F}_{avg} \Delta t = \vec{F}_{net,avg} \Delta t \quad (4)$$

without calculus but knowing the average forces acting during some time interval Δt . The net impulse imparted to a system of objects during some time interval is equal to the change in linear momentum of that system of objects during that time interval,

$$\vec{I}_{net} = \Delta \vec{p}_{sys} = \vec{p}_{final} - \vec{p}_{initial} = \sum_n m_n (\vec{v}_{final_n} - \vec{v}_{initial_n}) \quad (5)$$

where the momentum of the system is just the sum of the momenta of each of the system's parts. Remember, momentum is a *vector* and must therefore be added together taking directions into account. For one dimensional motions, this may be done by using plus and minus signs to indicate the two possible directions.

Now, suppose we know that the net, external force acting on a system of objects is zero. Equation 5 shows that the resulting net impulse imparted to the system is zero, and thus the change in the system's momentum must also be zero, resulting in a constant linear momentum for the system of objects. This is the condition for *conservation of linear momentum* of a system,

$$\vec{I}_{net} = 0 = \Delta \vec{p}_{sys} \Rightarrow \vec{p}_{final} = \vec{p}_{initial} \quad (6)$$

While the linear momentum of the system must remain unchanged, the linear momentum of the individual parts of the system may change. If one part of the system *gains* some amount of momentum, the rest of the system must *lose* that same amount of momentum. This result follows directly from Newton's 3rd law. When two members of a system exert forces on each other, these forces must be of the same magnitude and in opposite directions over the same time interval. Thus the impulse imparted to each object must be of equal magnitude but opposite direction. The momentum gained in one part of the system is lost in the other part, thus leaving the total momentum of the system unchanged. The only way the momentum of the system can change is by having a nonzero net impulse imparted by some entity outside of the system – an external force. The impulse-momentum relationship *always* holds true. Also, if the net impulse imparted to a system is zero, the linear momentum of that system is conserved.

Momentum is not the only quantity associated with the motion of a mass. A mass m moving with speed v also has *kinetic energy*,

$$\text{Kinetic Energy} = KE = \frac{mv^2}{2} \quad (7)$$

Momentum is a vector quantity, but kinetic energy is a *scalar* quantity. KE has no direction associated with it. Is kinetic energy also a conserved quantity for a system if the net external force acting on that system is zero? Well, kinetic energy changes if there is net work done on a system. Remember, Newton's 3rd law requires the impulse imparted to two interacting members of a system to be of equal magnitude and opposite directions thus causing the net impulse to be zero. However, Newton's 3rd law does *not* imply that each object will do the same amount of

work on each other. As a result the net work done may not be zero, even if there is no external force acting on the system, and the kinetic energy may change. In a collision, if kinetic energy is conserved we call that an *elastic* collision. In this case, the net work done on the system is zero. Only in a truly elastic collision will the kinetic energy of a system be conserved. In other cases, the kinetic energy will decrease due to negative net work being done on the system. These collisions are called *inelastic*. The greater the fraction of kinetic energy lost, the more inelastic the collision. The maximum fraction of kinetic energy that can be lost occurs when two objects collide and stick together permanently. Such collisions are called *perfectly inelastic*.

4 Procedure

4.1 Part 1. Determine the effective coefficient of friction

1. You must prepare the computer for data acquisition. Connect the two photo-gates and the force probe to the interface box and fire up the computer. The file *Linear Momentum parts 1&2* may be loaded for the setup parameters. The screen should show two data windows, each with a table. The first is titled “Time each gate blocked,” and has two columns. The first column tells the amount of time gate one is blocked, and the second does the same for the second gate. The other window is titled “Time between gates” and has a column that gives that time.
2. You should have two dynamics carts for your track. Measure and record the mass of each of these carts. Also measure the mass of the mass stick and record this value. Denote one of the carts as cart 1 and the other as cart 2. Using the calipers, measure and record the width of the PVC cylinder on each cart and record these values.
3. Your track should be placed on the lab bench and leveled as best as you can. Adjust the photo-gates so that they will be blocked by the PVC, and nothing else, as the carts move along the track. Be certain that the carts will not collide with any part of the photo-gate detectors. The support poles for the photo-gates should be about 60 cm apart.
4. Place one cart on the track and give it a push such that it first passes through gate 1 and then gate 2. If you are recording data, the tables should show the times for the three described intervals. Check to see if the times make rough physical sense. The cart must block gate 1 first. Think about whether the times for gate 1 and 2 should be the same or different and whether your times make sense.
5. Record these times. Repeat the measurement three more times at different initial cart speeds for four trials of: real slow, slow, medium, and fast. Record the measured times for all trials.
6. Now reverse the positions of gates 1 and 2. Repeat the previous measurements now with the same cart moving in the other direction.

4.2 Part 2. Collisions

1. You are going to have the carts collide in several different scenarios. For each collision you will want data such that you can determine the velocity of each cart before and after the collision. Remember, velocity is a vector and has both *magnitude* and *direction*. You will need both. The magnitude may be determined by knowing the time interval for which the cart blocked a photo-gate. Make sure you also have data which allows you to determine the

direction in which the cart moved. You may delete the table which read the time interval between gate readings.

2. For each of the ten collisions you will record two times for each cart. The time interval for which it blocked a photo-gate *before* colliding and a time interval for which it blocked a photo-gate *after* colliding. If the cart was initially at rest, denote this in some manner. Try to insure that the before times are measured as shortly before the collision as possible and that the after times are measured as shortly after the collision as possible by adjusting the positions of the photogates for each collision. The carts have magnets attached to one end so that they may “collide” elastically. There is Velcro on the other end so that they will stick together when they collide. For your first set of collisions the carts should collide with their magnet ends such that they will bounce off of each other.

Collision #1: Cart 2 at rest between gates, Cart 1 moves toward it and collides.

Collision #2: Cart 1 at rest between gates, Cart 2 with stick moves toward it and collides.

Collision #3: Cart 2 with stick at rest between gates, Cart 1 moves toward it and collides.

Collisions #4,#5,#6: The same as #1, #2, and #3 except the carts should collide with their Velcro ends touching such that they will now stick together. Again, if the initial speed of cart 1 is too high they may not stick together, so don’t get carried away.

Collision #7: Both carts with no stick moving toward each other and collide. Use the magnetic ends or, if you don’t have magnets, the plunger, so that the carts will bounce off of each other.

Collision #8: Repeat #7, but the mass stick is on Cart 2.

Collision #9: Compress the plunger on one cart and lock it in place. Center the carts on the track with the plunger up against the other cart. Tap the release on the top of the cart so the the plunger pushes out against the other cart. The initial velocity for each cart will be zero.

Collision #10: Repeat #9 but add the mass stick to cart 2.

4.3 Part 3. Impulse imparted by external force

1. For this section you will only need one cart, one photo-gate, and the force sensor. Remove gate 2 and load settings from the file *Linear Momentum part 3*. You should have one table for photo-gate readings and a graph of the force probe reading as a function of time. A spring should be connected to the force probe. The force probe will need to be held at the end of the track with the spring pointing back along the track. Place the photo-gate in front of the probe spring such that a cart may pass through the gate completely just before contacting the spring. You should use your hand to hold the force probe in position so that it doesn’t move.
2. Click “Record” and zero the force probe while it is on the track. Stop acquisition, delete data, and then click “Record” again while holding the force probe stationary. Give the cart a push such that it will pass through the gate, hit the spring, and bounce back through the gate. Stop the data acquisition and look at the resulting times and force probe graph while the cart was in contact with the spring.
3. You can determine the area under the force vs time graph by highlighting just the region of graph you would like to analyze. To do this, click on the fourth icon from the left along the

top of the graph window. A shaded area should appear on the graph which you can move with the mouse to position over the peak. If you click on the upper left icon the highlighted area will be zoomed into. Adjust the area until you have just the peak and then click on the fifth icon, which looks like a curve with shading under it. This will give a small table with the area under the curve. This area corresponds to the impulse imparted by the spring on the cart. Record time 1, time 2, and the area under the graph. Note: do not cause the spring to become *entirely* compressed during the collision – the resulting spike in your force graph makes the extracted impulse value inaccurate.

4. You can delete the last run by clicking on the icon at the bottom of the window by that name. Acquire data again, but this time push the cart through several times at different speeds. You will want data for at least 10 trials at a range of initial cart speeds. It will be convenient to just record another 10 or so trials at once and then analyze them consecutively. Record the appropriate data for each trial.

5 Calculations

5.1 Part 1. Determine effective coefficient of friction

1. For each of the first four trials, calculate the change in linear momentum for the cart during the interval of time it took to pass between the two photo-gates. Assume the cart is traveling in the positive direction. Since you now know $\Delta\vec{p}$ and Δt , use equation 5 to determine the net force acting on the cart as it moved along the track for each trial. Determine the mean and associated error in the mean for these four values.
2. Repeat these calculations for the four trials with the cart moving in the other direction on the track, again, assuming the cart is travelling in the positive direction. Check to see if the two results agree with each other? Should they?
3. Average these two mean values to determine the average frictional force acting on the cart. Determine an effective coefficient of friction between the cart and track.

5.2 Part 2. Collisions

1. You have 10 collisions to consider. For each collision determine the momentum of each cart both before and after the collision and the total linear momentum of the system both before and after the collision. If a cart was initially at rest, its initial linear momentum is zero.
2. List all these results, in tabular form, giving initial and final momentum of each cart, as well as the linear momentum of the system both before and after the collision.
3. Construct a graph plotting the linear momentum of the system after the collision on the y axis and the linear momentum of the system before the collision on the x axis. Determine the equation of the best fit line for this graph. Compare the slope and y intercept of your best fit line with the values predicted by conservation of linear momentum.

5.3 Part 3. External impulse

1. For each trial, calculate the change in velocity for the cart between the final time it passes through the photo-gate and the initial time it passes through the photo-gate. List these

results, for each trial, in a table along with the impulse imparted to the cart by the spring, as determined by the area under the force curve.

2. Construct a graph of Impulse imparted (as measured from your force graphs) vs change in velocity using the data from all trials. Using linear regression, determine the best fit straight line to this data. What should the slope and y intercept of this line be? If you don't know what the slope and y intercept should be, look at equation 5 for help. Compare your results with these expected theoretical values. Are they in agreement?

6 Questions

1. For argument's sake, let's say the slope of your graph from part 2 was not in agreement with one and all measurements were done and recorded correctly. If the slope was less than one, what would that mean? If the slope was greater than one, what would that mean? Explain.
2. For collisions 2 and 5 in part 2, calculate the system's kinetic energy both before and after the collision. Was kinetic energy conserved in these collisions? What type of collisions are these in terms of the last paragraph in the theory section? Explain.
3. Using your results from part 1 and knowledge of speeds and distances, estimate the impulse imparted to the cart by friction during the time interval of interest in part 3. Show your work and give a final answer. Whether or not this result is consistent with the results of your graph for part 3 may be an item to consider.

Linear Momentum Data Sheet

Part 1 - Effective coefficient of friction

Object	Mass	PVC Width
Units		
Cart 1		
Cart 2		

Mass Stick mass: _____

Data for cart moving in one direction on the track.

Trial	Gate 1 Time	Gate 2 Time	Time Interval
Units			
1			
2			
3			
4			

Data for cart moving in the other direction on the track.

Trial	Gate 1 Time	Gate 2 Time	Time Interval
Units			
1			
2			
3			
4			

Linear Momentum Data Sheet

Part 2 - Collisions

Trial	Mass 1	Mass 2	Time ₁ bef.	Time ₂ bef	Time ₁ aft	Time ₂ aft
Units						
1						
2						
3						
4						
5						
6						
7						
8						
9						
10						

LINEAR MOMENTUM DATA SHEET

Part 3 - External Impulse

Trial	Time 1	Time 2	Area under Curve
Units			
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			
11			
12			
13			
14			
15			