Graphical Analysis

1 Object

To become familiar with some principles of graphical analysis for use in later laboratories.

2 Apparatus

A simple pendulum, consisting of a stand, some string, and a mass. Also a stopwatch.

3 Theory

We should all remember the generic equation for a straight line:

$$y = mx + b \tag{1}$$

which says that the *dependent variable* y is proportional to the *independent variable* x with a *constant of proportionality* m (which is the *slope* of the line) and the *y-intercept* is b. The graph of this generic form is shown in figure 1 below.

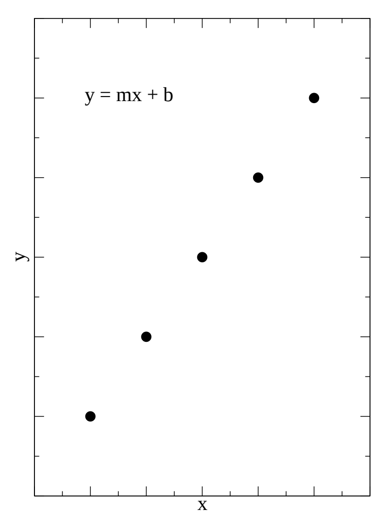

Figure 1: The generic graph of the straight line equation $y = mx + b$.

Any time a graph is a straight line, then this generic form $y = mx + b$ is applicable. Our goal in this lab will be to study some graphical forms and deduce how to re-plot them as a straight line. Then we will apply this to data taken from a simple pendulum.

The graphical form for $y = 1/x$ is shown plotted below on the left as y vs. x. This form is called *hyperbolic*. The graph on the right is a re-plot of y vs. the *quantity* $(1/x)$, and since we can write the equation above now as $y = 1 \cdot (1/x) + 0$, we see that y is proportional to the quantity $(1/x)$, and when compared to the generic form, should be a straight line.

$$\begin{aligned} y &= m \cdot x + b \\ y &= 1 \cdot (1/x) + 0 \end{aligned}$$

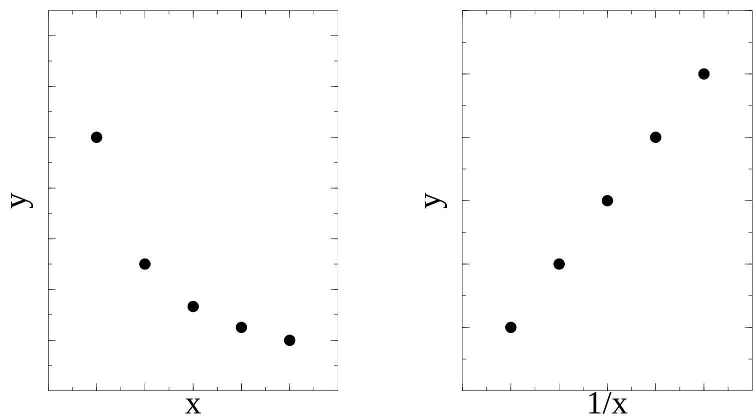

Figure 2: A plot of y vs. x for the equation $y = 1/x$ on the left; a re-plot of y vs. the quantity $(1/x)$ for the same equation yielding a straight line (right).

The graph of $y = 3x^2$ looks parabolic, as shown in figure 3. If we re-write the equation as $y = 3(x^2) + 0$, we see the generic $y = mx + b$ form with y now proportional to the quantity (x^2). Therefore a plot of y versus the quantity (x^2) should be a straight line of slope 3 and intercept 0.

The equation $y = 2x^{1/2} + 1$ plots as y versus x as shown in figure 4 on the left. If we re-write this equation as $y = 2(x^{1/2}) + 1$, we see that plotting the quantity $x^{1/2}$ as the dependent variable gives a straight line of slope 2 and intercept 1.

3.1 Logarithmic Plots

Logarithmic plots can be of great use for determining power law dependencies in mathematical relationships. For example, let's assume that a hidden mathematical relationship is

$$t = A \cdot l^{1/3} \tag{2}$$

Now, if we take the ln of both sides of this equation, we get

$$\ln t = \frac{1}{3}(\ln l) + \ln A \tag{3}$$

If one looks carefully at this equation, and considers the $\ln t$ as "y" and the $\ln l$ as "x" one recognizes this as nothing more than the old $y = mx + b$ form for a straight line. Notice that the slope is

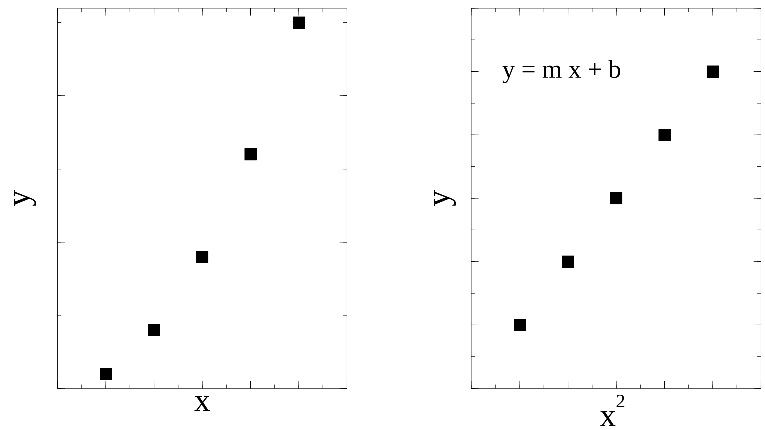

Figure 3: A plot of y vs. x for the equation $y = 3x^2$ on the left; a re-plot of y vs. the quantity (x^2) for the same equation yielding a straight line (right) of slope 3.

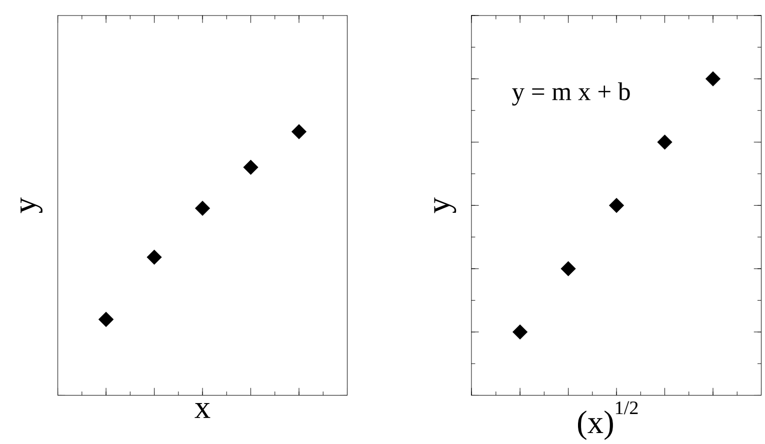

Figure 4: A plot of y vs. x for the equation $y = 2(x^{1/2}) + 1$ on the left. A re-plot of y vs. the quantity $x^{1/2}$ for the same equation yielding a straight line (right).

the power (in this case 1/3) that the variable is raised to – therefore with this graphing technique there is no guess work like the method used above because the number for the power is directly found by doing a linear regression to the data and reading off the slope of the line. Note also that the intercept is $\ln A$, the natural log of the constant. From this one can find the constant which multiplies out front in equation 2.

One more example is for equations of the form

$$y = Be^{-Cs} \tag{4}$$

where B and C are constants. If we now take the natural log of both sides of this equation we get

$$\ln y = -C \cdot (s) + \ln B \tag{5}$$

This once again has the $y = mx + b$ form, where $\ln y$ is "y" and s is "x" and the slope is $-C$. This type of graphing is useful where exponential relationships are expected, like in nuclear decay and in chemical reaction rates.

4 Procedure

Using the pendulum and an electronic stopwatch, you will time ten oscillations of the pendulum with a certain length of string and a fixed amount of mass on the end. Make sure that you use the protractor and always pull the mass through an angle of 5° at the start of every trial. Do 3 trials for a each length of the string, and make sure to do lengths of 10, 20, and 30 cm . Then you will change the length of the string and repeat, keeping the mass and angle the same. You will do this for a total of six (6) different lengths and all at the same mass and angle. Notice that we are varying only *one* possible variable (length) and keeping all others (mass, amplitude) fixed in order to see the effect of just the one being varied.

5 Calculations

1. Calculate the average time for the 10 oscillations for each length, and then calculate the time for one oscillation by dividing this average by 10. Helpful tip: you should have a neat table including all raw data for length and times, average measured time, and average time for one oscillation in the data and results section of your lab report.

2. Make a plot of the period (T) as the dependent variable (vertical) versus the length (l) as the independent variable (horizontal) to see how the period *depends* on the length. (This graph would be described as plotting period vs length.)

3. This plot should not look very linear, thus indicating that the period is not directly proportional to the length of a pendulum. As described in the handout, now create a plot of $ln(T)$ vs $ln(l)$. Do this by using *Excel* to make adjacent columns with $ln(l)$ values in one column and corresponding values for $ln(T)$ in the adjacent one, then graphing. This graph should look linear so perform a regression analysis to help determine the exponent for the power law relationship that best describes the relationship between period and length. This power should be either an integer or some fractional integer. It's unlikely that the actual relationship is that period is directly proportional to length raised to the 1.82457 power. Remember, the

slope and y-intercept of your best fit line are ranges. You want to find out what reasonable power is within the experimental range for the slope of the ln-ln graph.

4. Replot the data, by raising either period or length to some power, such that you will have a linear graph. Perform a linear regression, determining the slope and y-intercept of the best fit line to the data.

5. At this point, you can get from either the TA or the instructor, the theoretical values describing the relationship between the period and length of a pendulum. Use these given values to determine expected values for the slope and y-intercept of the best fit lines for each of your linear graphs (the last two). Helpful tip: the accepted values are not the same for the two graphs. As described in the handout, however, they are related. Include four-line-summaries for both the slope and y-intercept for each of your last two graphs. These summaries are your comparisons between theory and experiment. These summaries are where we will be checking for appropriate rounding of values.

6. In your results section, for *each* of the graphs you have been asked to do, tell us what the graph tells you about the relationship between the variables you plotted. That is, for the ln-ln graph, for example, why is it a straight line, and why are the slope and intercept what they are? This should be related to the underlying mathematical relationship between the variables.

7. At the conclusion of your results section, clearly articulate, based on your experiment, what your best estimate of the relationship between the period and length of a pendulum is and adequately defend your claim.

6 Questions

1. Sketch what a plot of the equation $y = x^3$ would look like. Do the same for $y = (x)^{\frac{1}{3}}$.

2. Other than using the log-log procedure, how would you re-plot to get straight lines for the above two equations?

3. What mathematical equation (for how the period of a pendulum depends on its length) results from your graphical analysis?

4. What would be the physical significance of your T vs. l graph going through the origin?

5. Can you tell from looking at your T vs. l graph that it is not linear? What if you were to only look at a section (small) of it – is this section obviously not linear? Explain.

7 Graphical Analysis Data Table

Length (cm)	Time 1 (s)	Time 2 (s)	Time 3 (s)	Avg/10 (s)
10				
20				
30				
100				

Mass: