1-Dim. Kinematics, Freefall

Object

To look at kinematics, with constant acceleration, in one dimension and experimentally determine the acceleration due to gravity. In addition, to review the analytic skills/techniques of the first two labs and better understand the connection between the graphs and the theoretical relationships between the variables graphed.

Apparatus

Electronic Timer, Meterstick, Steel Ball, Computer.

Theory

We know if an object is freely falling towards the ground, subject to no forces other than gravity, that it will fall with a constant acceleration directed downward equal to the acceleration of gravity: $9.8 \text{ m/s}^2$. The one-dimensional kinematic equations, assuming constant acceleration, describe the relationships between position, velocity, acceleration and time for this object.

\begin{align}
    x & = x_o + v_o t + \frac{1}{2} a t^2 \\
    v & = v_o + at \\
    a & = 9.8 \frac{m}{s^2} \text{ downward} \\
    v^2 & = v_o^2 + 2a(x - x_o)
\end{align}

These are the theoretically expected relationships involving position, velocity, acceleration and time. You will use these to perform calculations and to determine theoretically expected values for the slope and y-intercept of graphs you will make.

Notice, if an object starts from rest, $v_o = 0$ and moves distance $d$ in time $t$, equation 1.1 can be solved for $a$ and used to calculate the acceleration, assumed to be constant, of the object.

\begin{equation}
    a = \frac{2d}{t^2}
\end{equation}
Procedure

Freefall timing

1. Orient the Electronic Freefall Timer setup so that the ball will be released, fall some distance \( d \) and then strike the stop pad. Measure and record this distance \( d \) that the ball will fall. Use the timer setup to measure the amount of time it takes the ball to fall this distance, starting from rest. Repeat this time measurement 4 more times for a total of 5 time measurements.

2. Repeat this process for 9 other values of \( d \). These values should be uniformly spread out ranging from 2.0 meters or more down to 20 cm. Notice, you’ll be measuring times for 50 different drops.

Calculations

Direct Calculations:

1. For each distance, calculate the mean, standard deviation, standard error and associated error for the 5 time measurements. Put all your original data and calculated results into a easily read, well organized and labeled table. Note, for convenience in calculations to come, you will want to organize your data tables such that the distances are listed in order from smallest to largest. Add an additional column, labeled "Time Result" where you list the properly rounded result, value +/- unc, for your best estimate of the amount of time it took the ball to fall the corresponding distance from rest. Raw data and calculated values should be separated by a blank column whenever possible.

2. For each distance, use equation 1.5 to calculate the acceleration of the ball while it fell. You should use your measured distance and your best estimate for the time to fall that distance. The calculated value should be displayed in an additional column in your data and results table.

3. Calculate the mean, standard deviation, standard error and associated error for these 10 determined values for \( a \). All unrounded, calculated values should be listed and labeled. In a four line summary, compare your experimental result for \( a \) with the expected value for freefall.

4. In a new table, create two columns one with the mean time to fall and the next containing the corresponding distance for each of your 10 setups. These 10 distances and corresponding times define 9 separate time intervals. Create two more columns, one in which you calculate the average, or center, time for each of the 9 intervals and in the next calculate the average velocity for the ball during each of the 9 intervals using equation 1.6. We’ll assume that the average velocity of the ball during any interval is approximately equal to the instantaneous velocity of the ball at the middle of the interval timewise. This assumption should be true for constant acceleration motion.

\[
v_{avg} = \frac{(x_{n+1} - x_n)}{(t_{n+1} - t_n)} \quad \text{and} \quad t_{center} = \frac{(t_{n+1} + t_n)}{2} \quad (1.6)
\]

5. Your 9 calculated velocities and corresponding times now define 8 intervals over which you can calculate the average acceleration of the ball. Add two additional columns in which you calculate the center time of each of these 8 intervals and the average acceleration using equation 1.7.

\[
a_{avg} = \frac{(v_{n+1} - v_n)}{(t_{n+1} - t_n)} \quad (1.7)
\]
6. Since the acceleration should be constant, you would ideally expect all 8 of these values to be the same; 9.8 m/s². For this set of 8 values, calculate the mean, standard deviation, standard error and associated error. Compare this result with the accepted value for the acceleration due to gravity and check for agreement.

**Graphical Analysis:**

7. You are to make a graph of d vs \( t^n \) (n is an integer) using your data such that you expected a straight line. Looking at equation 1.1 and or equation 1.5, determine the appropriate value for the integer n and then make this graph using your data. Perform a linear regression to determine the slope and y-intercept of the best fit line, paste these coefficients to your graph and draw in the best fit line. Use equation 1.1 and or equation 1.5 to determine the theoretically expected values for the slope and y-intercept of this best fit line. In four-line summaries, compare your experimental results with these theoretically expected values.

8. Make a graph of v vs t using your data for the 9 intervals over which you calculated the average velocity of the ball. Perform a linear regression to find the best fit line. Use the appropriate kinematic equation to determine the theoretically expected values for the slope and y-intercept of this graph. Compare your experimental results with these theoretically expected values.

9. Make a graph of a vs t using your data for the 8 intervals over which you calculated the average acceleration of the ball. Perform a linear regression to find the best fit line. Use the appropriate kinematic equation to determine the theoretically expected values for the slope and y-intercept of this graph. Compare your experimental results with these theoretically expected ones.

10. Make a graph of \( t^2 \) vs d. Perform a linear regression to find the best fit line. Use the appropriate kinematic equation to determine the theoretically expected values for the slope and y-intercept of this graph. Compare your experimental results with these theoretically expected ones.

11. Make a graph of \( d^4 \) vs \( t^8 \). Perform a linear regression to find the best fit line. Use the appropriate kinematic equation to determine the theoretically expected values for the slope and y-intercept of this graph. Compare your experimental results with these theoretically expected ones.

**Optional Additional Analysis If Instructor Requests:**

12. Repeat calculation steps 4 and 5 using every other data point rather than adjacent data points. In other words, let points 1 and 3 define an interval, then 2 and 4, then 3 and 5,... You will only have 8 intervals over which you calculate velocities. Doing the same, you will then only have 6 intervals over which you calculate accelerations.

13. Repeat calculation steps 8 and 9 for these results. In addition, comment on how the results for these graphs compare with the results of graphs from the original steps 8 and 9.
Questions

1. If you had data for velocity and corresponding distance travelled and you made a graph of $d^3$ vs $v^n$ where $n$ is an integer such that the graph would be expected to be a straight line, what value of $n$ should be used and what would be the theoretically expected values for the slope and y intercept? Explain your reasoning and clearly show how you determined the expected values for the slope and y intercept.

2. Under what conditions would you not expect the y-intercept of your velocity vs time graph to be zero even though the object is freely falling? What physical quantity would the value of this non-zero intercept correspond to?

3. Consider your experimental results for acceleration from steps 3 and 6. Were these results in agreement with each other? How did the uncertainties in the two results compare with each other? Explain the cause/source of any substantial differences in the magnitude of these uncertainties even though the same raw data was used for each set of calculations.
KINEMATICS FREEFALL TIMER DATA SHEET

Freefall Electronic Timer

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