Newton's Second Law

1 Object

To investigate, understand and verify the relationship between an object's acceleration and the net force acting on that object as well as further understand the physical quantity acceleration.

2 Apparatus

Computer with Capstone software, motion detector, PVC pipe, low friction cart, track, meter stick.

3 Theory

A force is an interaction, between two objects, which attempts to change the motion of each object. A force is a *vector*, a quantity which is described by both a magnitude and a direction. The net force acting on an object at a particular point in time is the vector sum of all the individual forces acting on that object at that time. Two important things to note about the net force; first, the net force is not necessarily a "force" as described above. Secondly, the net force depends on the forces acting ON the particular object of interest. We do *not* concern ourselves with the forces the object of interest produces on *other* objects. Newton's second law relates the net force acting on an object to that object's acceleration

$$\vec{F}_{net} = \sum \vec{F}_{ind} = m\vec{a} \tag{1}$$

where $\vec{a}$ is the acceleration of the object and m is the object's mass. A useful visual tool when applying Newton's 2nd law is a free body diagram. A free body diagram yields a quick summary of all the forces acting on a particular object. For the purposes of determining only the acceleration of an object, a sufficient free body diagram is a dot, representing the object, and then an arrow drawn from that dot outward to represent each force acting on the object. It is the vector sum of these arrows (vectors) which is the net force acting on the object. You will investigate how Newton's 2nd law applies to three different experimental situations.

3.1 Part A: Cart on an inclined track

Consider a cart moving on a track, inclined at some angle θ, without friction. A free body diagram for this cart is illustrated in the figure below. It consists of two forces acting on the cart, the weight force due to the earth's gravity and the normal force due to the track. We know the cart does not accelerate perpendicular to the track, any acceleration must be parallel to the direction of the track. Applying Newton's 2nd law to the cart in these two component directions gives

$$\sum \vec{F}_\perp = m\vec{a}_\perp \qquad \sum \vec{F}_{||} = m\vec{a}_{||}$$
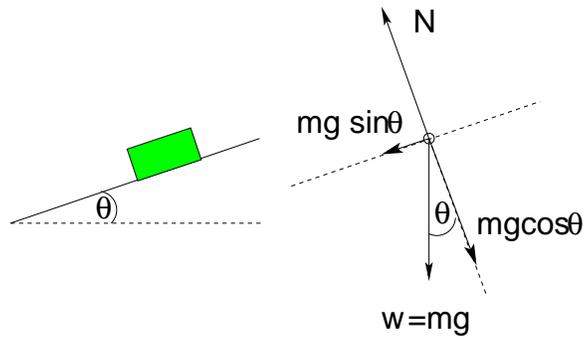
$$N - mg\cos\theta = 0 \qquad\qquad mg\sin\theta = ma \qquad\qquad (2)$$
$$N = mg\cos\theta \qquad\qquad a = g\sin\theta$$

For a track inclined to a particular angle θ and a cart of mass m, equation 2 allows us to determine the magnitude of the normal force acting on the cart due to the track and the expected acceleration of the cart as it moves along the track. Notice, no matter where the cart is on the track, as long as there are no forces other than those shown in the free body diagram, the cart will have the same acceleration, independent of the cart's location or velocity.

3.2 Part B: Object on an incline with friction
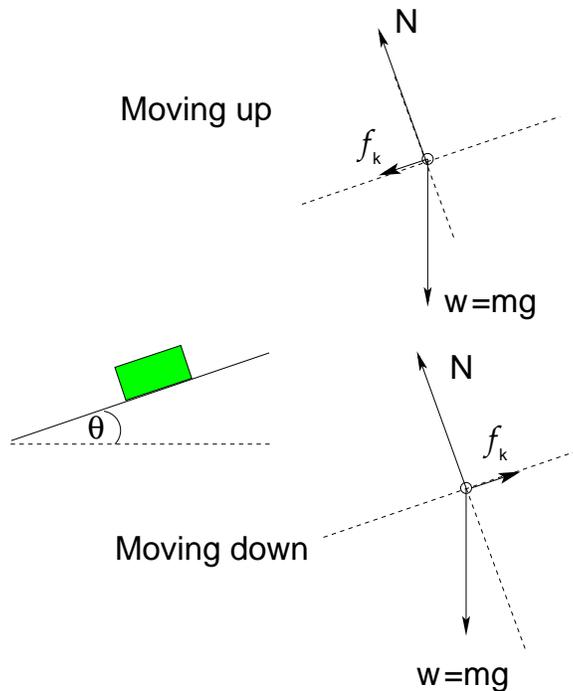
Consider an object moving on a track inclined at some angle θ to the horizontal as before, only now let there be a nonzero coefficient of kinetic friction, μ_k. Now the additional force of friction must

be included when using Newton's 2nd law to determine the acceleration of the cart. The direction of the kinetic friction force will be such that it opposes the relative motion between the object and the track. Since we'll assume the track stays stationary, the kinetic friction force on the object by the track will act in the opposite direction of the object's motion. As a result, if the object moves up the incline, the frictional force will point back down the incline. If the object moves down the incline, friction will point back up the incline. In both cases, the normal force would still remain the same, the forces perpendicular to the incline are still the same as if there was no friction regardless of the direction of motion. Hence, the magnitude of the frictional force will be the same whether the object moves up or down the incline. However, since the direction of that frictional force will differ for motion up or down the incline, we will expect to see different accelerations for motion up and motion down the incline. Applying Newton's 2nd law for the up and down motion of the cart would yield

$$\sum \vec{F}_{\|_{up}} = m\vec{a}_{\|_{up}} \qquad\qquad \sum \vec{F}_{\|_{down}} = m\vec{a}_{\|_{down}}$$
$$mg\sin\theta + f_{kin} = ma_{\|_{up}} \qquad\qquad mg\sin\theta - f_{kin} = ma_{\|_{down}} \qquad (3)$$

The acceleration going up the incline should be larger in magnitude than the acceleration going back down the incline. The difference in the accelerations up or down the incline is related to the frictional force as

$$\frac{m\left(a_{up} - a_{down}\right)}{2} = f_{kin} = \mu_k mg\cos\theta \qquad (4)$$

Notice, if the coefficient of kinetic friction is zero, then the accelerations up and down the incline should be the same.

4 Procedure

4.1 Part A: Cart on an inclined track

1. Position the track at your bench such that it is inclined at an angle of approximately $8°$ to the horizontal. Attach the motion detector to the higher end of the track so that the motion detector faces down the track, parallel to the track's surface.

2. The computer must be set up to acquire data. Turn on the computer as well as the *Pasco 850* interface box. Connect the motion detector to the interface box. It plugs into the two digital input ports. The yellow input must go in the leftmost digital port used. Open the program *Pasco capstone* by clicking on the icon on the desktop. You may setup the computer to take data by loading settings from a saved file. Click on the icon for opening files and select the file *Second Law*. The result should be three graphs for the position, velocity, and acceleration of the cart as a function of time. Clicking on the "Record" button will cause data to be taken (clicking on the same button, now labeled "Stop," will cause data collection to cease). While taking data, check that the motion detector is working by putting your hand about 6 inches in front of the detector and moving your hand toward the far end of the track at a fairly constant slow speed. Do the resulting graphs make sense? While doing this check to see that there are no other objects that the motion detector sees other than what is on the track. When convinced the detector and computer are working, click on the stop button.

3. Place the cart on the track so that the plunger is facing away from the motion detector. Give the cart a push so that it moves up the incline and then comes back. Experiment until you discover how hard to push the cart so that it goes most of the way up, but comes no closer than 30 cm from the motion sensor. Now you are ready to take data. Click on the "Record" button, give the cart a push so that it goes up the track and then comes down. Stop the cart and then click on the stop button. Make sure you move your hand out of the way so that the motion detector only "sees" the cart while it moves up and down the track. Adjust the time interval shown on the graphs until you see just the period during which the cart moved up and down the track. Do the graphs make sense to you? Repeat the experiment until you have graphs that appear reasonable. You may want to have the instructor or TA check your graphs. You may determine the average acceleration of the cart in two ways. First, on the acceleration graph, select the icon that looks like a highlighter. A box should appear on the graph, and you should move it until it contains the data you want to use. Clicking on the summation symbol should cause the mean value of the data in the box to appear. Second, fit a straight line to the velocity graph. The slope of this line is the average acceleration of the cart as it moved on the track. To do this, again click on the highlighter and box in the data of interest. Then click on the icon that looks like a red line fitting points, and select Linear. A box should appear that gives slope and intercept for that region. Record these two values and print copies of the graphs for your report. For the time when the cart was moving up the track, when was the velocity of the cart zero? When was the acceleration of the cart zero? Using a meter stick, measure the heights of the $70\,cm$ and $170\,cm$ marks on the track above the table. Use this data, calculate the angle of inclination of the track. Record these three values.

4. Repeat step 3 for angles of approximately 10, 12, 5, and 3 degrees. For each angle record the information used to calculate the angle, the angle of inclination, and the acceleration of the cart determined by each method.

5. Do you notice anything peculiar about the graphs for an angle of inclination of 3 degrees? Repeat again at an angle of about 1 degree and look at the results. In addition to the previous data, treat the time intervals when the cart moves up the track and down the track *separately*, determine and record the average acceleration of the cart for each interval.

6. Use the balance scale to determine the mass of the cart and record this value on the data sheet.

4.2   Part B: PVC cylinder on an incline.

1. Adjust the track such that it is inclined at an angle of about 8 degrees. Measure this angle of inclination as accurately as possible and record it.

2. Before trying to take data, it will be useful to increase the rate at which the equipment takes data. To do this find near the bottom of the window a button that says Motion Sensor II, and just to the right of this is the area where you change the data acquisition rate. Change it to $40\,Hz$.

3. For this part of the lab, you will be sliding the PVC cylinder on the track and measuring the

acceleration with the motion detector. First, give the PVC cylinder a push so that it slides UP the incline. Acquire motion detector data as before and determine the acceleration of the cylinder from the velocity graph. Repeat this four more times so that you will have five measurements of the acceleration of the cylinder when it is sliding up the track.

4. Repeat step 2, but now only have the cylinder sliding DOWN the incline instead. Record the five independent measurements of the acceleration of the PVC cylinder when sliding DOWN the incline.

5 Calculations

5.1 Part A: Cart on an inclined track

1. Make a graph of the cart's acceleration as a function of $\sin\theta$ using the data from the 6 trials in part A. What should this graph look like? What does equation 2 say this graph should look like? Use a linear regression program to determine the best fit line to the data. It is important that the program you use also allows you to determine the associated error for both the slope and y-intercept of the best fit line. The slope of this line should be equal to $g = 9.8\,m/s^2$. The y-intercept should be zero. Are the results from your best fit line in agreement with the expected results?

2. From the data at an angle of inclination of about 1 degree, estimate the magnitude of the frictional force acting on the cart. Calculate an effective coefficient of rolling friction, $\mu_{eff} = \frac{f}{N}$ where f is the magnitude of the frictional force and N is the magnitude of the normal force.

5.2 Part B: PVC cylinder on an incline.

1. Calculate the mean, standard deviation, standard error and associated error using your five measurements for the acceleration of the PVC cylinder moving UP the incline.

2. Repeat for the PVC cylinder moving DOWN the incline.

3. Calculate the magnitude of the kinetic frictional force acting on the PVC cylinder when sliding on the track. Also calculate the coefficient of kinetic friction between the PVC cylinder and the track.

4. Use the associated errors in the acceleration values to determine the uncertainty in your calculated value for the coefficient of kinetic friction.

$$\delta\mu_k = \frac{\sqrt{(\delta a_{up})^2 + (\delta a_{down})^2}}{2g\cos\theta} \tag{5}$$

6   Questions

1. You took data with the PVC cylinder at an angle near 8 degrees. If instead, you had taken data at an angle of 12 degrees, would the magnitude of the acceleration of the cylinder when moving up the incline had been larger, smaller or the same? What about when it was moving down the incline? Explain both answers.

2. When the cylinder was sliding on the track at an angle close to 8 degrees, which of the following has a larger magnitude; the kinetic friction force or the parallel component of the weight force acting on the cylinder? Explain how you can tell without making any calculations at all.

3. Why does the acceleration of the cart going up and down the incline only look significantly different at very small angles of inclination?

7 Newton's 2nd Law Data Sheet

7.1 Part A - Cart on inclined track

Trial	height$_{70}$	height$_{170}$	$\theta_{calculated}$	a_{slope}	a_{mean}
Units					
1					
2					
3					
4					
5					

Trial	height$_{70}$	height$_{170}$	$\theta_{calculated}$	$a_{slopeup}$	a_{meanup}	$a_{slopedown}$	$a_{meandown}$
Units							
6							

$$m_{cart}: \underline{\hspace{3cm}}$$

7.2 Part B - PVC cylinder on Incline

Trial	$a_1\ (m/s^2)$	$a_2\ (m/s^2)$	$a_3\ (m/s^2)$	$a_4\ (m/s^2)$	$a_5\ (m/s^2)$	$a_{mean}\ (m/s^2)$	$\delta a\ (m/s^2)$
UP							
DOWN							

$$\text{PVC mass:} \underline{\hspace{3cm}}$$