Equilibrium

1 Object

To determine the mass of unknown objects by utilizing the known force requirements of an equilibrium situation.

2 Apparatus

Force table, masses, mass pans, metal loop, pulleys, strings, two unknown masses, balance, force probe, computer with software.

3 Theory

There are two conditions on an object that must be met for it to be in equilibrium. They are:

$$\sum \vec{F} = 0 \qquad and \qquad \sum \vec{\tau} = 0 \tag{1}$$

Since $\vec{F} = m\vec{a}$ and $\vec{\tau} = I\vec{\alpha}$, this implies that an object in equilibrium has neither linear nor angular acceleration, and thus constant linear and angular velocities. If these velocities are zero, they will remain so for as long as the net force and net torque remain zero. This is called *static equilibrium*. If either the linear velocity or angular velocity is non-zero yet the net force and torque are zero, the situation is referred to as *dynamic equilibrium*.

3.1 Part 1

If only a point mass is considered, it will have no dimensions and thus can have no torques associated with it. The condition for equilibrium then reduces to

$$\sum \vec{F} = 0 \tag{2}$$

In general, since $\vec{F}$, the force, is a vector, it will have both x and y components given by

$$F_x = F\cos(\theta) \quad and \quad F_y = F\sin(\theta) \tag{3}$$

where θ is the angle the force vector makes with the positive x axis.

In this particular experiment the forces are produced by a string connected horizontally to a metal ring and then extended over a pulley and connected to a mass at the other end. The mass hanging from the end of the string will also be in equilibrium so equation 2 must also hold for it as well. There are only two forces acting on this mass, the tension force due to the connected string and the gravitational force down on the mass, $W = mg$. If the net force is zero, these two forces must be in opposite directions and have equal magnitudes. As a result, the resulting tension in the string connected to the hanging mass must be equal to the gravitational force on that mass

$$T = W = mg \tag{4}$$

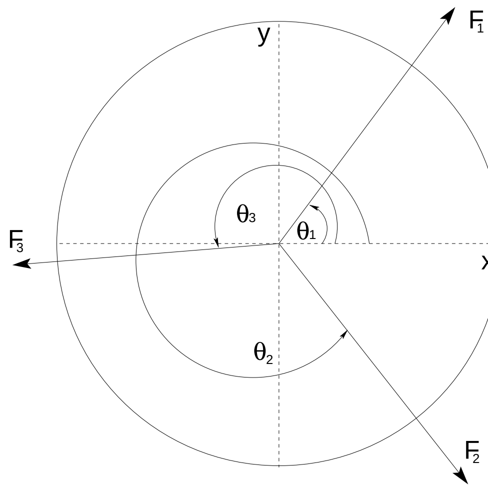
In most cases there is more than one force acting on a given object (can an object with only one force acting on it be in equilibrium?), and the condition for equilibrium, Equation 2, can be written out as

$$\sum \vec{F} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots = 0 \tag{5}$$

Each of these vectors may be written in component form using equation 3. Equation 5 can then be written in component form:

$$F_{1x} + F_{2x} + F_{3x} + \ldots = 0 \quad and \quad F_{1y} + F_{2y} + F_{3y} + \ldots = 0 \tag{6}$$

where the component of every force acting on the object should be included. In this lab, we will



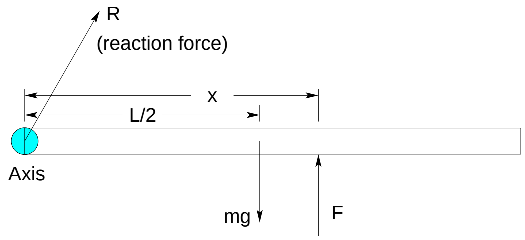
be considering the horizontal forces acting on the metal ring. Each string exerts a tension force on the metal ring, which we know is equal to the gravitational force on the mass suspended from that string. Combining this knowledge with equation 6 and defining the positive x axis as $\theta = 0$ (see the figure above), we see that

$$\begin{aligned} F_{Tx} &= m_1 g \cos(\theta_1) + m_2 g \cos(\theta_2) + m_3 g \cos(\theta_3) + \ldots = 0 \\ F_{Ty} &= m_1 g \sin(\theta_1) + m_2 g \sin(\theta_2) + m_3 g \sin(\theta_3) + \ldots = 0 \end{aligned} \tag{7}$$

F_{Tx} and F_{Ty} represent the component of the net force in the x and y directions, respectively. If the angle for each force is known and all the masses are known except for one, either of the relationships from equation 7 may be solved for the value of the unknown mass.

3.2 Part 2

What if the object under consideration is not a point mass? In that case, there may be different *torques* exerted on the object due to the different individual forces acting on it. However, if the object is in equilibrium, the conditions of equation 1 must still hold true, and the different torques, when added together as vectors, must equal zero. Let us consider a long board of mass m and length L, supported, parallel to the ground, at one end by a hinge and at some distance x away from that hinge from underneath by some vertical force F. This situation is pictured below in the figure. In accordance with equation 1, the net force acting on the board must be zero if the board is in equilibrium. The reaction forces on the board by the hinge in the vertical and horizontal directions may be whatever magnitude they must be to insure the net force on the board is zero, as long as the structural integrity of the hinge isn't reached. However, the net torque on the board (about *any* and *every* point) must also be zero. Let us consider the point at the end of the board where the hinge is connected. Since the reaction forces on the board due to the hinge act on a line passing directly through this point, these forces can exert no torque on the board about the hinge



point. This leaves two forces which may exert torques on the board, the weight force on the board due to the earth, and the vertical force up on the board from underneath. In accordance with equation 1, the torques due to these forces must be of equal magnitudes and in opposite directions so that when adding these torques together we will have a net torque of zero and the board will be in equilibrium. We can find the torque on an object about some point by an external force $\vec{F}$ acting on the object a distance r away from that point using the following relationship,

$$\vec{\tau} = \vec{r} \times \vec{F} \qquad |\vec{\tau}| = \tau = rF\sin(\varphi) \tag{8}$$

where $\vec{r}$ is the vector extending from the chosen point to where the force is exerted on the object, $\vec{F}$ is the force under consideration, and φ is the angle between these two vectors. Notice if φ is 0 or 180 degrees then the torque due to the force $\vec{F}$ is zero. The right hand rule for the vector cross product of $\vec{r}$ into $\vec{F}$ will give the direction of the torque due to the force $\vec{F}$. You may do this by pointing the fingers of your right hand in the direction of $\vec{r}$, curling them towards the direction of $\vec{F}$, and then your thumb will point in the direction of the torque due to the force $\vec{F}$. Inspection of the picture, or use of the right hand rule, should convince you that the weight force on the board and the vertical force from underneath ($\vec{F}$) are indeed trying to rotate the board about the hinge point in opposite directions. Setting the magnitudes of these two forces equal and using equation 8 yields

$$xF\sin(90°) = \frac{L}{2}mg\sin(90°) \quad or \quad xF = \frac{mgL}{2} \tag{9}$$

where g is the magnitude of the gravitational field.

4 Procedure

4.1 Part 1. Force Table

1. Set up the force table as described below. For part one you will need three pulleys and a string connected to the center metal loop and passing over each pulley. Connect mass hangers to two of the strings and one of your unknown masses to the third string. The unknown mass should roughly be larger than $100\,g$ but less than $300\,g$. Place the string connected to the unknown mass at the angle specified for trial 1. Add mass to one hanger until the total mass, including the 50 hanger, is the specified amount and place that pulley at the listed angle. Now, vary the attached mass and angular location of the third string until the metal loop is in a state of equilibrium. Equilibrium can be recognized when the metal loop is placed centered around the center rod in the table and it remains there. Record the total mass attached to the third string and the angular location of the pulley. Be sure and use the 1 and 2 gram masses so that you are measuring the third mass to the nearest gram.

2. Once you have achieved equilibrium and recorded the values for the mass suspended from the third string and its angular location, approximate the uncertainty in these values. Experi-

mentally determine how much you can change the angle of the third pulley before the ring is noticeably out of equilibrium. Also determine by how much you can change the mass before the ring is noticeably out of equilibrium. Record these values on the data sheet.

3. Repeat steps 1 and 2 for 4 other sets of pulley locations. The mass and pulley angle location for the second string are not specified for the last two trials. You are free to vary those however you desire. Show some creativity and make these situations as different from each other and the previous trials as possible. You would be well advised to not place the unknown mass at an angle which is an integer multiple of 90 degrees.

4. Now, using the triple beam balance scale, measure and record the mass of your unknown object.

4.2 Part 2. Force Probe

1. At your bench, you should have a long board with two eyelet screws at one end. There should also be a horizontal, thin bar attached to a stationary vertical bar. Thread the horizontal bar through the two eyelets and you have a long board, hinged at one end. Orient things so that the board is across the bench with most things out of the way.

2. The computer must be set up to acquire data. Turn on the computer as well as the *Science Workshop 500* interface box. Connect the force probe to the first analog port. Open the program "Data Studio". You may setup the computer to take data by loading settings from a saved file. Click on "Open Activity" and then open the file *Equilibrium.* The result should be a digital readout for the force being exerted by the force probe. Click "Start" button to take data. Set the force probe on the table vertically so that the hook end is pointing straight up. Zero the force probe by pushing the "tare" button on its side. The digital readout should now read 0.000. If it doesn't then re-zero the probe.

3. Place the force probe under the board near the end opposite the hinge and adjust the hinge height until the board is completely level. You are now ready to take data. Record the distance x from the hinge end at which the force probe is touching the board and record F the magnitude of the force exerted by the force probe.

4. Raise the unhinged end of the board slightly and move the probe closer to the hinged end, recheck that the probe is zeroed properly, lower the board back onto the probe, check that the board is level and again record x and F.

5. Repeat this for 13 more points along the board spaced between the unhinged end and no closer than $10\,cm$ to the hinged end. DO NOT EXCEED 50 N for the FORCE PROBE READING!!!

6. Remove the board from the horizontal bar; measure and record the board's mass and length.

5 Calculations

5.1 Part 1. Force Table

1. For each of the five sets of data using three strings, determine the value of the first unknown mass from looking at each set of components, equation 7. This means you will calculate the mass of this unknown object 10 times. Remember, the components of a vector are still vectors

themselves and it is important to incorporate the direction of these vectors when adding and subtracting. Since all the x components lie along the x axis, there are only two possible directions which can be described using plus or minus signs. Remember to make sure the signs are correct.

2. Calculate a mean and error for the experimental value of the mass of your first unknown object. Is this value consistent with the result of the direct measurement using the balance scale?

5.2 Part 2. Force Probe

1. Construct a graph of F vs x using your data from part 2. What does this graph look like? Is this what you expected?

2. Construct a graph of F vs $1/x$ using your data from part 2. What does this graph look like? Is this what you expected?

3. Using a linear regression program, find the best fit straight line to your graph of F vs $1/x$. Some manipulation of equation 9 results in a prediction that the y intercept of this line should be zero and the slope of this line should be $mgL/2$. Using your measured values for m and L, calculate a predicted value for this slope and determine if the predicted values for the slope and intercept of this graph are in agreement with the experimental results.

6 Questions

1. Once equilibrium is established on the force table, would doubling the masses require new angles? What if you instead, added $100\,g$ to every string? Explain.

2. Is the error in your mean value for the mass of the unknown consistent with your estimations of the uncertainty in the measured values θ and m for each trial? Explain.

3. The mass of the string is not considered in this experiment. Why? Under what conditions would you take into account the string's mass?

4. If instead of 3 strings you only had two, one with your first unknown attached, what would the angle between the strings be? How much mass would be attached to the second string. Prove your answer mathematically using equation 2.

7 Equilibrium Data Sheet

Part 1 - Equilibrium with three pulleys

Trial	Mass 1	Theta 1	Unknown θ	Mass 2	Theta 2	δM2 (g)	$\delta\theta$2 (deg)
Units							
1	100	60	160				
2	150	225	345				
3	125	145	80				
4							
5							

Unknown mass measured using the triple beam balance

Measured Unknown Mass (g)

Equilibrium Data Sheet – cont'd

Part 2 - Force Probe

Trial	x	F
Units		
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		
11		
12		
13		
14		
15		

Measured Board Length	Measured Board Mass 2