THE BALLISTIC PENDULUM

Object

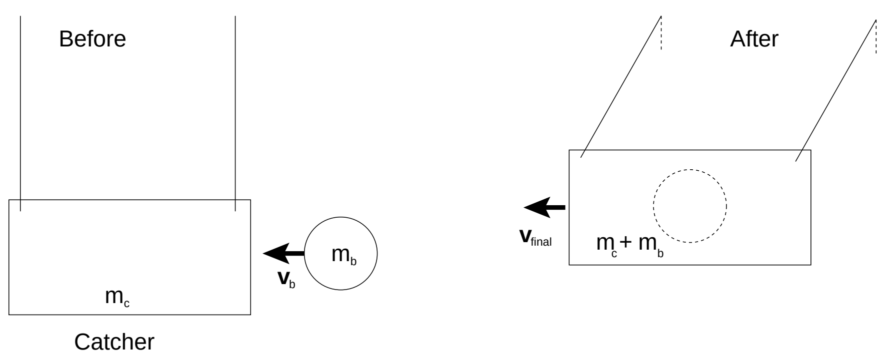
To apply the ideas of conservation of momentum and conservation of energy, when appropriate, to the ballistic pendulum experiment. To experimentally measure the velocity of a fired ball using three methods, maximum vertical height, cart on track and the change in height of the ballistic pendulum, and see how well these results agree.

Apparatus

A projectile launcher, a ballistic pendulum, meterstick and caliper, a balance, steel ball, string, dynamics cart, dynamics track and photogate.

Theory

We know that if there is no external net force acting on a system, linear momentum is a conserved quantity in collisions. We also know that for elastic collisions, kinetic energy is also a conserved quantity. How can these ideas be applied to the case of a ballistic pendulum, where a ball is fired at some initial speed into a pendulum catcher which is connected to a string and swings like a pendulum? We will neglect the effects of air resistance and internal friction in the system.



Let us first consider the first part of this motion, the collision of the moving ball with the stationary pendulum. After colliding, the ball is stuck in the pendulum and both are moving with the same velocity. We know this to be a perfectly inelastic collision which means that the kinetic energy in the system will not be conserved. The momentum of the system, however, will be conserved. Conservation of momentum may be applied assuming all motion is in the horizontal direction:

$$\mathbf{p}_{initial} = \mathbf{p}_{final} \quad or \quad m_b\mathbf{v}_{ob} + m_c\mathbf{v}_{oc} = (m_b + m_c)\,\mathbf{v}_{final} \tag{1.1}$$

where m_b refers to the mass of the ball and m_c refers to the mass of the catcher. Since, for this particular case, the initial velocity of the catcher is zero, we can solve for the final velocity of the system:

$$\mathbf{v}_{final} = \frac{m_b\mathbf{v}_{ob}}{(m_b + m_c)} \tag{1.2}$$

Now we have a catcher, with ball stuck inside, moving horizontally at some speed. Since the catcher is attached to a string of fixed length, it will behave as a pendulum, swinging up in an arc of some fixed radius. We are neglecting air resistance which leaves gravity and the tension force due to the string as the only forces acting on the pendulum. Gravity is a conservative force which means work done by it may be described in a potential energy term. Since the Tension force due to the string always acts perpendicular to the motion of the pendulum, this force does zero work at all times. Since there is no work done on the system, catcher and gravitational field, by nonconservative or external forces, the total mechanical energy of this system must be conserved. There is kinetic energy stored in the motion of the masses and there is gravitational potential energy due to the vertical position of these masses. Conservation of mechanical energy means:

$$U_{grav_{init.}} + KE_{init.} = U_{grav_{final}} + KE_{final} \tag{1.3}$$

The gravitational potential energy due the position of a mass is just mgy where y is the vertical position of that mass m. If we call the initial location of the catcher the vertical origin, then the initial gravitational potential energy is zero. As the pendulum swings up it loses kinetic energy and an equal amount of potential energy is stored in the gravitational field. When the pendulum reaches its highest vertical position, the kinetic energy of the system is zero, $KE_{final} = 0$. Substitution into equation 1.3 yields:

$$\frac{(m_b \ + \ m_c)\, v_{final}{}^2}{2} \ = \ (m_b \ + \ m_c)\, gy \tag{1.4}$$

where y is the final vertical position of the pendulum and v_{final} is the final speed of the catcher after the initial collision with the ball. We know this speed from equation 1.2 so substitution will give us a theoretical equation for the final vertical position of the ballistic pendulum:

$$y \ = \ \frac{(m_b \mathbf{v}_{ob})^2}{2g\,(m_b \ + \ m_c)^2} \quad \Rightarrow \quad \mathbf{v}_{\mathrm{ob}} \ = \ \sqrt{2\mathrm{gy}}\left(\frac{\mathrm{m_b} \ + \ \mathrm{m_c}}{\mathrm{m_b}}\right) \tag{1.5}$$

Does this result make physical sense in the limits where the mass of either the ball or the catcher is zero?

Procedure

Part A. Determining the initial speed of the launched ball via max height measurement

1. What happens energy-wise as the ball is launched horizontally from the launcher? Potential energy is stored in the compressed spring, as the spring decompresses, some of that energy is converted into kinetic energy stored in the balls motion. The ball then leaves the launcher with some amount of kinetic energy due to its mass and nonzero speed. You will use energy considerations to determine the speed with which the steel ball leaves the launcher. If you fire the ball straight up, the kinetic energy of the ball is transformed into gravitational potential energy due to the ball's vertical position until the ball reaches its maximum vertical position when all the kinetic energy has become gravitational potential energy.

2. Orient the launcher such that the steel ball may be fired straight up. When fired, the ball should go straight up and come straight back down to the launcher, or at least awfully close to it.

3. Measure and record the vertical position of the ball when the launcher first stops pushing on the ball. A picture of the ball in this position is illustrated on the side of the launcher.

4. Fire the ball and estimate how far above the launcher the ball goes before stopping. Use masking tape to attach a meterstick to the wall, centered about the approximate highest vertical location of the ball, such that the ball will move upwards directly in front of the meterstick. Make sure the meterstick is as vertical as possible.

5. Compress the ball fully into the launcher. Have one partner positioned such that t heir eyes are about

level with the highest vertical position of the ball. Have the other partner fire the ball. Record the highest position of the ball. You should be able to measure this to at least the nearest .5 cm, maybe even better than that.

6. Repeat this procedure to measure the maximum height a total of 5 times.

7. Measure and record the mass of the steel ball.

Part B. Cart on a track.

1. For this part of the lab, you will need to attach the catcher to the top of your dynamics cart and place the cart on a leveled track. You will orient the launcher so that it fires the ball straight in the horizontal and is positioned so that the ball will be fired directly into the catcher along the line of the dynamics track. The cart will then move down the track. It is important that there be a gap between the launcher and the catcher so that the launcher will not touch the catcher or contact it before the catcher moves.

2. Place a photogate such that the beam will be blocked as the cart with catcher attached moves past it.

3. Fire the ball into the catcher and record the time interval for which the photobeam is blocked.

4. Repeat for a total of 5 measurements.

5. Record the length of the object which actually blocked the photogate beam. Also record the combined mass of the catcher and cart assembly.

Part C. The ballistic pendulum.

1. Measure and record the mass of the catcher. Attach the catcher to the upper connection plate such that the catcher is level and free to swing. The length of string between the catcher and the top connection panel should be approximately 40 cm.

2. Mount the launcher on the tabletop and adjust it such that it again will fire a ball horizontally. Adjust the height of the top panel such that the steel ball may again be launched directly into the catcher. If the ball is fired into the catcher, it should swing up and out some distance. Again, make sure there is a gap between the catcher and the launcher so there will be no contact during the firing.

3. A fifth string should be hanging free from the catcher. The free end of this string should be attached to the Velcro assembly on the launcher apparatus.

4. Fire the ball into the catcher and notice how much string is pulled through the Velcro. Reduce the length of string between the launcher and catcher just slightly. If the ball was to be fired again, just a small extra length of string would now be pulled through the Velcro.

5. Fire the ball into the catcher. You may now determine the final vertical position of the catcher after catching the ball. To do this, by hand, move the pendulum along its path until the fifth string is again taut. With the pendulum in this position, measure its final vertical position using the meterstick. Lower the pendulum back to its equilibrium position and measure its initial vertical position. Record these values as well as the length of the pendulum.

6. Repeat this procedure four more times using the steel ball.

7. Adjust the ballistic pendulum so that the length of string between the catcher and top connection point is approximately 90 cm. Repeat steps 6-7 with the steel ball and again determine the initial and final vertical

position of the pendulum.

Calculations

Part A.
1. We'll assume mechanical energy is conserved as the ball moves upward. Use equation 1.3 to determine the initial speed of the steel ball as it leaves the launcher for each of your measured final vertical position values.
2. Calculate the mean, standard deviation, standard error and associated error for these initial speed values. You now have your result for the launch speed of the ball from part A. List this result, properly rounded.

Part B.
1. Calculate the velocity of the cart/catcher/ball as it moved on the track for each of your measured times.

2. Use conservation of momentum to calculate the launch velocity of the ball for each trial.

3. Calculate the mean, standard deviation, standard error and associated error for this set of launch speed values. You now have your result for the launch speed of the ball from part B. List this result, properly rounded.

4. Check for agreement between this result and the corresponding experimental result from Part A.

Part C.
1. Using the experimental values for y for your first ballistic pendulum, calculate the final speed of the catcher after colliding with the ball but before it started to move upwards. You are applying conservation of mechanical energy using equation 1.4.

2. Now, for each of these trials, calculate the initial velocity of the ball before it was fired into the catcher using conservation of linear momentum via equation 1.2.

3. Calculate the mean, standard deviation, standard error and associated error for this set of launch speed values. You now have your result for the launch speed of the ball from the first ballistic pendulum in Part C. List this result, properly rounded.

4. Repeat steps 1-3 for the data from your second ballistic pendulum length.

5. Compare these two experimental values for the launch speed with each other and those of parts A and B. Are they in agreement? You should have 6 comparisons in total for the lab.

Questions

1. In which portions of this lab is mechanical energy not conserved? When is linear momentum not conserved? What is responsible for these quantities not being conserved if they aren't? You may consider the idealized cases where we neglect air resistance…

2. Which of the experimental methods, Part A B or C, do you think is the most accurate determination of the launch velocity? Justify your answer.

3. If the ball launcher was placed on a frictionless surface and then fired with the ball going into the catcher, would the ballistic pendulum rise higher, not as high, or the same distance? Explain.

BALLISTIC PENDULUM DATA SHEET

Part A - Determine initial speed of launched ball

Ball	Steel
Mass	
Initial Vert Position	
Final Vert. Position 1	
Final Vert. Position 2	
Final Vert. Position 3	
Final Vert. Position 4	
Final Vert. Position 5	

Part B - Cart on the track.

Catcher/cart Mass	
Blocker length	

	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Time Interval					

Part C - The ballistic pendulum

Catcher Mass	

	$\ell \approx 40$cm	$\ell \approx 90$cm
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Initial Vert. Pos.		
Final Vert. Pos. 1		
Final Vert. Pos. 2		
Final Vert. Pos. 3		
Final Vert. Pos. 4		
Final Vert. Pos. 5		