Archimedes' Principle

1 Object

To determine the density of objects by using Archimedes' principle and to compare with a density measured directly.

2 Apparatus

Assorted masses, balance, beakers, graduated cylinders, masking tape, measuring stick, Pasco 850 Universal Interface and force probe, unknowns, vernier caliper, water.

3 Theory

In this experiment we will concern ourselves with the density of various objects. The density ρ of an object is defined as its mass m divided by its volume V, or

$$\rho = \frac{m}{V} \tag{1}$$

Archimedes was the first to give a valid mechanistic explanation for the observation that an object appears to weigh less in water than air. The basic theoretical principle applicable is his explanation of why this weight loss takes place.

Consider a fluid (gas or liquid) and consider in more detail a section of that fluid shown below. If

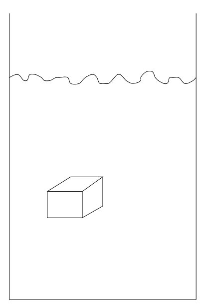

Figure 1: A small volume of fluid.

the fluid is at rest, this section will be in equilibrium. Specifically, that is, the forces upward will balance the forces downward. Figure 2 shows these in more detail. There will be a downward force F_D on the upper face due to the pressure, which in turn is due to the weight of the fluid above this face. Likewise there will be an upward force F_U due to the pressure on the lower face. A third force arises due to the weight of this volume of fluid. All of these forces must balance since the section of fluid does not move ($F = ma = 0$). Mathematically this is written as

$$F_U = F_D + W_{fluid} \tag{2}$$

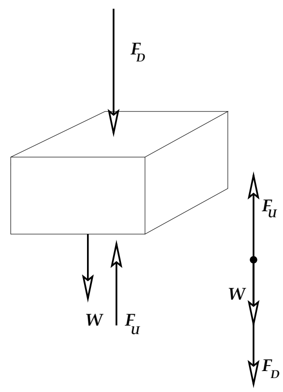

Figure 2: A small volume of fluid and associated forces as in the text.

For the sake of simplicity let us consider a cubic segment with each side having an area A. Pressure P is defined as force per unit area, thus

$$F_U = P_U A$$
$$F_D = P_D A$$

here P_U is the pressure at the lower face and P_D the pressure at the upper face. Further, the pressure on a fluid increases with depth so that $P_U > P_D$. Equation 2 then becomes

$$P_U A = P_D A + W \qquad \left(NOTE : P_U > P_D \; since \; P_U = P_D + \frac{W}{A}\right)$$

The next step consists of replacing the section of fluid by another substance. It will experience the same pressure forces which when combined will be in an upward direction. These two forces combined are called the *buoyant* force F_B

$$F_B = F_U - F_D$$

But from the above considerations

$$F_B = W_{fluid} \tag{3}$$

where W_{fluid} is the weight of the fluid that would occupy the same volume as the object does now. This is Archimedes' principle and in words it states that any object totally or partially immersed in a fluid is buoyed up by a force equal to the weight of the fluid displaced by the object.

A variation on the expression for the weight of an object is stated

$$W = mg = \rho V g \tag{4}$$

where ρ is the mass density of the object, V is its volume, and g is the acceleration due to gravity. The buoyant force on an object can be determined by measuring its weight in air, W_{air}, and its "effective weight" in water, F_{water}. This "effective weight" in water is the net vertical force on the submerged object and is the difference between W_{air} and F_B. In terms of F_B one has:

$$F_B = W_{air} - F_{water}$$

Equation 3 says this also equals the weight of the fluid displaced, W_{fluid}. The volume of the object, V_{unk}, is equal to the volume of the fluid displaced (the fluid it pushes out of the way), V_{fluid}. Hence we get

$$W_{air} - F_{water} = W_{fluid}$$

Both terms on the left of this expression will be directly measured on the force probe and one can solve for the volume of the unknown using equation 4 to obtain

$$V_{unk} = V_{fluid} = \frac{W_{air} - F_{water}}{\rho_{fluid} g} \tag{5}$$

where ρ_{fluid} is the density of the fluid which in this case is water with value $1.00\, g/cm^3$. The density of the object can then be determined from equations 1 and 5

$$\rho = \frac{m}{V_{unk}} = \frac{mg}{V_{unk} g} = \frac{W_{air}}{V_{unk} g} \tag{6}$$

Accepted densities for various materials are:

Material	Density (g/cm^3)
Al	2.70
Cu	8.93
Brass	8.44
Cork	0.22 - 0.26
Glass	2.60
Rubber	1.25
Steel	7.80
Wood	0.35 - 0.60

4 Procedure

1. Start the *PASCO Capstone* program on your computer, and load the file *archlab*. You should see a window that will display the force probe reading, and a graph of force (N) versus time (s).
2. You should have a total of ten objects for this lab. You will need to hang your objects from the force probe, and you will need to use something with which to hang it. Typically this will be a mass hanger, but need not be.
3. You will be required to get a force probe measurement for each object in two ways: once suspended in air, and once when the object is submerged in water. You will need to record each of these readings separately in the data table. For an **air reading**, you should first zero the force probe with the suspension device (mass hanger) on it by pressing *Start* and pushing the tare button on the side of the probe. For a **water reading**, zero the probe when the suspension device (not the probe) is under water by lifting the bucket up and then having your partner push the tare button. Once you have properly zeroed the probe, then you can put the object on the suspension device.

4. To take data once the object is suspended either in air or under water and the probe is properly zeroed, click *Record* on the program, and allow it to acquire data for about 10 seconds. What should the graph look like?

5. Click *Stop*, and then use the mouse to highlight the correct region of the graph, and use the value in the legend for mean as your force value in your data table. You will want to delete the data after each run.

6. Note that if your object would like to float then you will need to be a bit more creative in how you attach it to the force probe.

7. You should give your object a unique name in your data table in case you need to go back and check values.

8. Note that the value for the force probe in the numeric window will often vary by some small amount (while you acquire data). You can use this variation as an estimate for the error on the force probe.

9. You will need to determine the volume of each of your objects as well. There are two ways to do this. For a regularly shaped object, you should measure the appropriate dimensions of the object (*i.e.*, length, width, and height) for calculating the volume. For irregularly shaped objects, you will need to submerge them in water that is in a container with a volume scale on it. These should be provided in the lab. Record the dimensions or the volume in your data table as well.

10. Once you have taken both air and submerged readings for the first object, and done the volume measurement(s), you can remove it and put another object on the probe. Repeat the procedure for all of your objects.

5 Calculations

1. Using equation 1, calculate the density of the unknown using the measured mass and your found volume. Given the experimental procedure used, what is the error on this density?

2. Using the data from the force probe readings, calculate the density of each of the unknowns. You should use equation 5 to get the volume. The mass can be obtained by dividing the weight in air reading from part 2 by the acceleration due to gravity, $9.8\,m/s^2$. From our error estimates on the force probe readings, find an error estimate on the density.

   Compare the density results from calculations part 1 and part 2 for each object.

5.1 A note on Errors

You were given a handout on error propagation at the beginning of the semester. This is a quick reminder of some of the things stated therein – how to calculate the error on a result of a calculation when the numbers used in the calculation have errors. For example, in this lab equation 1 will be used to calculate density. The mass and volume will each have some errors δm and δV, respectively.

The error on the density is then given by:

$$\delta\rho = \rho\sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta V}{V}\right)^2}$$

A similar but more complicated process holds for the results from procedures 2 and 3. If we assume that the errors on W_{air} and F_{water} are about the same then $\delta V = \sqrt{2}\,\delta W$, where δW is the error on either force probe reading. It will not be twice the error since sometimes the errors on the two forces will at least partially cancel out. Then as above one gets

$$\delta\rho = \rho\sqrt{\left(\frac{\delta W}{W}\right)^2 + \left(\frac{\delta V}{V}\right)^2} = \sqrt{3}\,\rho\,\left(\frac{\delta W_{air}}{W_{air}}\right)$$

since $\delta V = \sqrt{2}\,\delta W$. Here we have explicitly used the error on the weight reading in air. This error will in reality probably be a bit too small since $\delta F_{water}/F_{water}$ is larger than $\delta W_{air}/W_{air}$ (since $W_{air} > F_{water}$). However, it is a good approximation at a realistic error.

Finally, when comparing two numbers and their associated errors – $N_1 \pm \delta N_1$ and $N_2 \pm \delta N_2$ – to see if they are equal, one might do the following. Let $C = N_1 - N_2$ and

$$\delta C = \sqrt{(\delta N_1)^2 + (\delta N_2)^2}$$

If the range $C \pm \delta C$ includes zero, then the two numbers are equal. This last process could be helpful when comparing the density results for procedure 1 and procedures 2 and 3.

6 Questions

1. Why should each unknown be completely submerged in part 3 of this experiment?
2. Which density value is more accurate? Why?
3. Could your equipment tell the difference between a copper and a brass object? Explain.

7 Archchimedes Principle data sheet

7.1 Part 1

Object Description		Mass of Object (g)	Dimension/Volume (cm^3)	Error on Vol. (cm^3)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				

7.2 Part 2

Object	Weight in Air (N)	Error on Weight (N)
1		
2		
3		
4		
5		
6		
7		
8		
9		
10		

7.3 Part 3

Object	Force reading in Water (N)	Error on force (N)	Buoyant Force (N)
1			
2			
3			
4			
5			
6			
7			
8			
9			
10			