Angular Momentum

1 Object

To verify conservation of angular momentum, determine the moment of inertia for various objects and look at the exchange of angular momentum in different situations.

2 Apparatus

A rotational dynamics air table, two steel disks, one aluminum disk, one plate assembly, two different steel hoops, a balance, meter sick, and computer with interface software.

3 Theory

While we may be quite familiar with linear momentum ($\vec{p}$) and the related quantities mass (m) and velocity ($\vec{v}$), the quantities angular momentum ($\vec{L}$), moment of inertia (I), and angular velocity ($\vec{\omega}$) are somewhat less familiar. Linear momentum is a physical quantity which describes the motion of the center of mass of an object, or a system of objects. Angular momentum is a physical quantity which describes the motion of an object, or a system of objects, rotating about a given axis. If you throw a stick, the motion of the center of mass of the stick moving across the room results in linear momentum, the stick spinning or rotating as it flies across the room results in angular momentum. Linear momentum, $\vec{p} = m\vec{v}$, is a useful quantity, in part, since if there is no external net force acting on a system, the total linear momentum of that system is a conserved quantity. Remember,

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} \qquad \vec{F}_{net} = 0 \Rightarrow \frac{d\vec{p}}{dt} = 0 \quad \Rightarrow \vec{p} = constant \tag{1}$$

A similar rule is true concerning angular momentum. If a system has no net *external* torque acting on it, the total angular momentum, $\vec{L} = I\vec{\omega}$, is conserved or constant.

$$\vec{\tau}_{net} = \frac{d\vec{L}}{dt} \qquad \vec{\tau}_{net} = 0 \quad \Rightarrow \quad \frac{d\vec{L}}{dt} = 0 \quad \Rightarrow \quad \vec{L} = constant \tag{2}$$

As a result, analogous to the usefulness of conservation of linear momentum when studying linear collisions, conservation of angular momentum is quite useful when studying situations involving rotating objects. We know if there is no external torque on a system then $\vec{L}_{initial} = \vec{L}_{final}$, always.

Notice, there is a great similarity between the relationships for linear and angular momentum resulting in a correlation between the relevant physical quantities. Just as $\vec{p}$ describes linear momentum, $\vec{L}$ describes angular momentum. Just as velocity, $\vec{v}$, describes the linear motion of an object, rotational velocity, $\vec{\omega}$, describes the rotational motion of an object about a given axis. The velocity of a point mass and its rotational velocity about an axis are related by

$$\vec{\omega} = \frac{\vec{r} \times \vec{v}}{r^2} \tag{3}$$

where $\vec{r}$ is the shortest vector extending from the axis to the location of the point mass. Just as mass, m, is a measure of an object's ability to resist changes in its motion, moment of inertia, I, is a measure of an object's ability to resist changes in its rotational motion about an axis. Since we must specify a given axis, the same object will most likely have a different moment of inertia about different axes. The moment of inertia of a point mass about a given axis is $I = mr^2$ where m is the mass and r is shortest distance from the axis to the point mass. Likewise, the moment of inertia of a system of point masses may be found by

$$I = \sum_{n=1}^{N} I_n = \sum_{n=1}^{\mathrm{N}} m_n r_n^2 \tag{4}$$

If instead we have a continuous object we may find its moment of inertia by considering it to be an infinite number of point masses dm and integrating

$$I = \int_{Vol} r^2\, dm \tag{5}$$

where the integral is over the *entire volume* of the object. Your book gives formulas for the moments of inertia of several geometric objects – hoops, disks, rods, spheres *etc.* – about different axes. The moment of inertia of a system of continuous objects may be found by summing the moments of inertia of the different objects. Of particular interest in this lab is the moment of inertia of a uniform disk of mass m, outer radius R_{out} and inner radius R_{in} about the axis of symmetry through the disk's center. The moment of inertia is given by

$$I = \frac{m(R_{out}^2 + R_{in}^2)}{2} \tag{6}$$

In an analogous manner, we may either determine an object's linear kinetic energy; energy stored in linear motion of a mass; or its rotational kinetic energy; energy stored in the rotational motion of a mass;

$$KE_{linear} = \frac{mv^2}{2} \qquad KE_{rot} = \frac{I\omega^2}{2} \tag{7}$$

Conservation of angular momentum means that the vector sum of all the angular momentum of each object in a system at some initial point in time will equal the vector sum of the angular momentum of each of those same objects at any later point in time,

$$\vec{L}_{initial} = \vec{L}_{final} \;\; or \;\; \sum_{n=1}^{N} I_n \vec{\omega}_{n_{initial}} = \sum_{n=1}^{N} I_n \vec{\omega}_{n_{final}} \tag{8}$$

We will make use of this relationship to verify the conservation of angular momentum in different situations by determining the angular momentum of a system both before and after some event. If there is no external torque, the angular momentum should always be the same. In a like manner, we can also check whether the kinetic energy of a system is conserved or not.

4 Procedure

CAUTION CAUTION CAUTION CAUTION

The surfaces of the steel and aluminum disks are very smooth so that friction may be minimized. If the surfaces are even slightly damaged, the disks are ruined. Please use great care when handling the disks. Only hold them by the outer edges, never on the flat surfaces which slide against one another. Take caution not to get these surfaces dirty or drop objects on the disks.

4.1 Part A. Conservation of Angular Momentum

1. The rotational dynamics air table should already be assembled and connected to an air supply via a pressure regulator. All required equipment should be present at the lab tables. Verify that the apparatus is level.

2. Measure and record the mass and radius of each disk.

3. Turn on the air supply, adjust the regulator to $12\,psi$.

4. Place the steel disk with the large hole in the center on the rotation device. Place the second steel disk on top and you are ready to take data. If the pin is placed in the small hole at the center of the top disk, a thin layer of air will be created between the disks allowing them to rotate quite free of frictional forces between their surfaces. There is a small air hose coming out the bottom of the apparatus with a clasp on it. If this hose is shut, then a layer of air is also created between the bottom disk and the apparatus so that there will be negligible friction between these surfaces as well. For part A, this hose should always be clasped shut. With the hose shut and the pin in place, both disks may be spun in any manner, independent of the other disks motion. When the pin is removed, the disks will "collide" and stick together.

5. Open *Pasco Capstone* and load the file "Angular Momentum" which has two graphs (one for each disk). Click on "Record" and spin first one disk and then the other so you can see which graph is for which disk. The graph tells you the angular frequency of each disk versus time.

6. Now, click stop and then start again and spin the top disk but not the bottom disk. Pull the pin so the disks collide and watch the graphs. By highlighting the data before and after the collision you can get average values for the angular velocity of each disk. Record the rotational frequency and direction of rotation for each disk before and both disks after the collision for the following scenarios with two steel disks:

   (a) Bottom disk spinning, top disk stationary.

   (b) Both disks spinning, in same direction, with different frequencies.

   (c) Both disks spinning, in opposite directions, with different frequencies.

   (d) Both disks spinning, in opposite directions, with about the same frequency.

7. Repeat the four cases using the aluminum disk on top.

4.2 Part B. Determining Moments of Inertia

1. We will now assume angular momentum is conserved in all collisions.

2. Place the steel disk on top and attach the plate assembly using a hollow screw and the small pulley spacer.
3. You have two steel hoops with small pins on one side. These pins should fit into the matching holes on the square plate now attached to the top disk. You will experimentally determine the moment of inertia of these hoops, both separately and combined, and compare this with the theoretical value.
4. Record the mass of each hoop as well as the outer and inner radii.
5. Place the small hoop on the square plate and the pin through the hollow screw in the center hole so that the top conglomeration will spin freely.
6. With the top disk, and attached objects, stationary, spin the bottom disk and then pull the pin so that the two disks will stick together. Record the initial frequency of the bottom disk (frequency of top disk is zero) and the final frequency of the whole assembly. Do this for 5 different initial bottom disk frequencies ranging from 50 to 600 marks/second.
7. Repeat this procedure with both the small and large hoops attached to the top disk.

5 Calculations

5.1 Part A.

1. Using equation 6, calculate and list the moment of inertia for each of the three disks used with the apparatus; bottom steel disk, top steel disk and top aluminum disk.
2. Use the data from part A to calculate the total angular momentum of the system, both before and after pulling the pin, using equation 8. Do this for each of the four cases with two steel disks and also for each of the four cases involving one steel and one aluminum disk. Make a table listing: case number, initial rotational frequency of each disk, final rotational frequency of the system, initial angular momentum of each disk, initial angular momentum of the system, final angular momentum of each disk, final angular momentum of the system, and the % difference (difference divided by the initial times 100) between $L_{i_{system}}$ and $L_{f_{system}}$.
3. Using the results from these eight cases, construct a graph with initial angular momentum of the system on the y axis and final angular momentum of the system on the x axis. Determine the equation of the best fit line and compare the results with those predicted by conservation of angular momentum.

NOTE: since you will have an ω in each term when calculating angular momenta, you do not need to convert the degrees per second frequencies into angular velocities in rad/s. You may use the frequency readings as if they were in rad/s for all of the lab. This will save you some calculations.

5.2 Part B.

1. Calculate and list theoretical values for the moment of inertia of the two hoops using measured values for R_{in}, R_{out}, m and equation 6.
2. You have five sets of data for each apparatus configuration. Calculate the initial angular momentum of the bottom disk for each of the five trials.
3. Make a graph of initial angular momentum vs final rotational velocity using these five points.

Determine the best fit line and compare the slope and y-intercept with theoretically expected values.
4. Use the equation of this line to extract a value for the moment of inertia of the hoop attached to the top disk. Calculate the % err between this value and the theoretically expected value.
You may assume the mass of the square plate and small pulley are negligible. Look at equations 8 and 4 to help figure out how to extract the moment of inertia values.
5. Repeat this procedure for all configurations of part B.
6. In a table, list the values for the moment of inertia for the attached object as extracted from the graphs, the theoretical values for these moments of inertia and the % error for all three sets of data.

NOTE: since you will have an ω in each term when calculating angular momentums, you do not need to convert the marks per second frequencies into angular velocities in radians per second. You may use the frequency readings as if they were in radians/second for all of part B. This will save you some calculations.

6 Questions

1. Is angular momentum always conserved? Is kinetic energy always conserved? Would you say the collisions in Part A were elastic or inelastic collisions? Explain where any missing kinetic energy went. Could you describe a possible case for part A where the kinetic energy of the system would be conserved?
2. How did the slopes of the three graphs in part B compare? If I gave you a $1.0\,ft$ long steel rod of diameter $1.0\,cm$, how would you directly attach it to the top disk so that you could repeat part B and get the smallest possible slope? How would you attach it to get the steepest possible slope?
3. You are sitting in the middle of a $6.0\,m$ diameter, freely rotating, giant turntable which is currently rotating at a rotational velocity of $8.5\,rad/s$. You think you are about to vomit so you stand up and start to walk straight out from the center toward the edge. As you walk toward the edge, does the turntable's rotational velocity increase, decrease, or remain the same? Explain your answer.

Angular Momentum Data Sheet

Part 1

Disk	Mass	Outer Radius	Inner Radius
units			
Bot. Steel			
Top Steel			
Alum.			

Top Disk	Bot Disk Init Freq	Top Disk Init Freq	Final Freq
S			
S			
S			
S			
A			
A			
A			
A			

Part 2

Object	Mass	Outer Radius	Inner Radius
units			
Small Hoop			
Large Hoop			

Small Hoop	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Init freq					
Final freq					

Big Hoop	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Init freq					
Final freq					

Both Hoops	Trial 1	Trial 2	Trial 3	Trial 4	Trial 5
Init freq					
Final freq					