

Acceleration due to Gravity

1 Object

To determine the acceleration due to gravity by different methods.

2 Apparatus

Balance, ball bearing, clamps, electric timers, meter stick, paper strips, precision pulley, ramps, weights, metal track.

3 Theory

According to Newton any two objects will attract one another if they both have mass. This attraction, F , is given by the formula

$$F = \frac{GM_1M_2}{r^2} \quad (1)$$

where M_1 and M_2 are the masses of the two objects and r is the distance between the centers of mass of the two. G is a constant of proportionality, $6.67 \times 10^{-11} Nm^2/kg^2$.

If one of these objects is the Earth (M_E) and the other is a relatively small object near the surface of the Earth (M_o), the distance between the two is essentially the radius of the Earth, R_E . Then equation 1 becomes

$$F = \frac{GM_E M_o}{R_E^2} \quad (2)$$

where M_E is the mass of the Earth ($5.983 \times 10^{24} kg$), R_E is its radius ($6.371 \times 10^6 m$) and M_o is the mass of the object.

If the object is in free fall, this is the only force acting on it and

$$F = M_o a = M_o g \quad (3)$$

where g is the acceleration due to the gravity of the Earth.

Combining the last two equations gives

$$M_o g = \frac{GM_E M_o}{R_E^2} \quad (4)$$

$$g = \frac{GM_E}{R_E^2} \quad (5)$$

or

$$g = 9.807 m/s^2 \quad (6)$$

In situations where the object is not in free fall it will still interact with the Earth and this will cause the object to have a weight, W , which is given by $M_o g$.

The weight of an object as well as the acceleration due to gravity are variables, but if one stays near the surface of the Earth both may be considered to be constant. In this experiment g will be determined by a number of methods.

3.1 Atwood's Machine

A common device used to calculate accelerations is Atwood's apparatus. In this case, it consists of two masses, m_1 and m_2 , attached by a string (assumed massless) which in turn is on a pulley. Schematically it looks like Fig. 1. Applying Newton's Second Law to each object (assuming

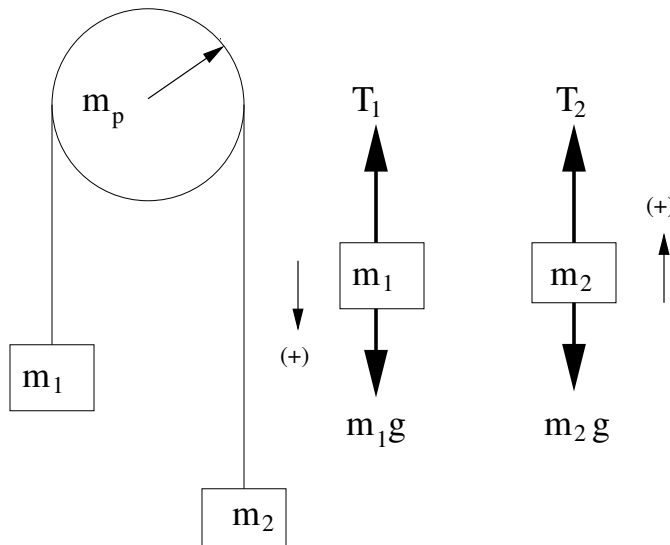


Figure 1: Atwood's machine (left) and free body diagrams for each mass (right) with forces labeled.

$m_1 > m_2$) one has

$$\begin{aligned} m_1 a &= m_1 g - T \\ m_2 a &= T - m_2 g \end{aligned} \quad (7)$$

The accelerations are equal since the objects are tied together, and it is customary to assume the tension equal and evenly distributed throughout a string. Solving the above equations for the acceleration due to gravity gives:

$$g = \left(\frac{m_1 + m_2}{m_1 - m_2} \right) a \quad (8)$$

If the acceleration a has an error associated with it, δa , then the corresponding error on g , δg , will be given by:

$$\delta g = \left(\frac{m_1 + m_2}{m_1 - m_2} \right) \delta a \quad (9)$$

The above discussion ignores the inertia of the pulley. If it is to be taken into account one must consider the rotational acceleration, α , caused by the string:

$$\tau = I\alpha \quad (10)$$

Where $I = \frac{1}{2}MR^2$ is the rotational inertia of a disk, $\alpha = a/R$ for the edge of the pulley, m_p is the mass of the pulley and R is its radius.

Now, however, the two tensions are different and equations 7 become

$$\begin{aligned} m_1 a &= m_1 g - T_1 \\ m_2 a &= T_2 - m_2 g \end{aligned} \quad (11)$$

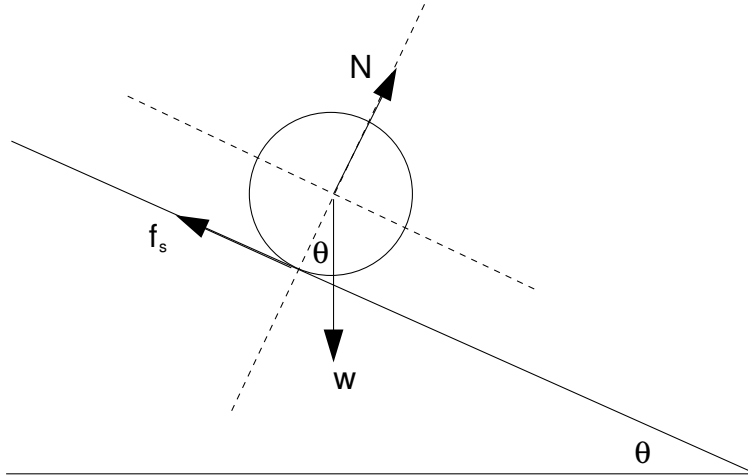


Figure 2: Galileo's ramp with a ball rolling down without slipping. The three forces *on* the ball are shown. The frictional force produces the torque about the center of the ball.

and these two tensions cause the pulley to rotate by producing torques.

The total torque, τ , produced by the strings is:

$$\tau = T_1 R - T_2 R \quad (12)$$

Equations 10 and 12 give

$$\tau = I\alpha = \frac{1}{2}m_p R^2 \frac{a}{R} = T_1 R - T_2 R \quad (13)$$

or

$$\frac{1}{2}m_p a = T_1 - T_2 \quad (14)$$

Combining equations 11 and 14 and solving for g yields

$$g = \left(\frac{m_1 + m_2 + 0.5m_p}{m_1 - m_2} \right) a \quad (15)$$

with an associated error on g of

$$\delta g = \left(\frac{m_1 + m_2 + 0.5m_p}{m_1 - m_2} \right) \delta a \quad (16)$$

3.2 Galileo's Ramp

Galileo measured the acceleration of objects as they rolled down a ramp. He did this instead of working with falling objects at least partially because the balls on ramps rolled much slower and were more easily observed. The acceleration of the ball on the ramp can be related to the acceleration due to gravity. Consider a ball rolling down the ramp shown in Figure 2. Using Newton's Second Law of motion, $F = ma$, in two dimensions gives: *parallel to incline*

$$W \sin \theta - f_s = ma \quad (17)$$

perpendicular to incline

$$N - W \cos \theta = 0 \quad (18)$$

(no motion and thus no acceleration \perp to incline)

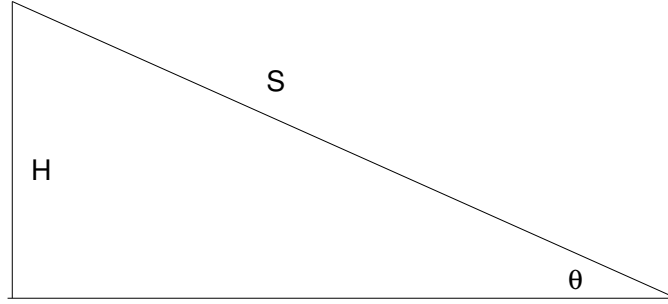


Figure 3: The relationship between H , S , and $\sin \theta$.

Applying the rotational equivalent of Newton's Second Law about the center of mass yields

$$\tau = I\alpha \quad (19)$$

$$f_s R = \left(\frac{2}{5}mR^2\right)\alpha \quad \left(I_{\text{sphere}} = \frac{2}{5}mR^2\right) \quad (20)$$

because only the friction force produces a torque. So

$$f_s = \frac{2}{5}mR\alpha \quad (21)$$

but for rolling motion $\alpha = a/R$. This leads to

$$f_s = \frac{2}{5}ma \quad (22)$$

Using this and $W = mg$, equation 17 becomes

$$\begin{aligned} mg \sin \theta - \frac{2}{5}ma &= ma \\ mg \sin \theta &= \frac{7}{5}ma \end{aligned} \quad (23)$$

Solving for g yields

$$\boxed{g = \left(\frac{7}{5 \sin \theta}\right) a} \quad (24)$$

$$\boxed{\delta g = \left(\frac{7}{5 \sin \theta}\right) \delta a} \quad (25)$$

where a is the acceleration of the ball down the plane and θ is the angle the plane makes with the horizontal. Here, $\sin \theta = \frac{H}{S}$, as shown in Figure 3, where S is the length of the ramp and H is the height of the track above the other end.

4 Procedure

Note on measuring acceleration: motion in one direction is given by $d = v_o t + \frac{1}{2}at^2$, where d is the distance the object moves, v_o is the initial velocity, a is the constant acceleration, and t is the elapsed time. If the object starts from rest, then $v_o = 0$, and:

$$d = \frac{1}{2}at^2 \quad (26)$$

Solving for a gives:

$$a = \frac{2d}{t^2} \quad \delta a = 2a \frac{\delta t}{t} \quad (27)$$

If d and t are known, then a can be calculated.

4.1 Atwood's Machine

Put a string over the pulley and attach the pulley firmly to a support apparatus. Attach known masses to both ends of the string and record the time necessary for the heavier mass to fall a known distance. No masses should come in contact with the pulley, or the floor. Information recorded should be the two masses (m_1 heavier), the mass of the pulley ($m_p = 114\text{ g}$ for the solid grey pulley; $m_p = 68\text{ g}$ for the orange spoked pulley), the distance traveled by the masses, and the time interval.

Repeat for a total of three (3) trials, and redo the trials for a total of three different mass combinations.

4.2 Galileo's Ramp

Note the ball that is to roll down the incline must be a solid ball. Set up the track so that it is at an incline. Record the vertical height (H) and the length of the track (S) for the angle θ calculation. Also record the distance d that the ball will traverse while being timed. Then record the time for the ball to travel this distance, starting from rest, for a total of three (3) times. Use total of three (3) different balls and repeat the trials.

Change the inclination of the track and the length d and repeat the procedure just outlined above.

5 Calculations

5.1 Atwood's Machine

Calculate the mean time t_{avg} and its associated error δt for the time of fall of the heavier mass. From these results, calculate the mean acceleration a_{avg} and its associated error δa using equations 27. Then calculate the mean value of the acceleration of gravity g and its associated error δg using equations 15,16. Compare your value of g to the accepted one.

5.2 Galileo's Ramp

Calculate, for each ball and track inclination, the mean time t_{avg} and its associated error δt for the ball to roll the distance d . Then calculate a_{avg} and δa as above. Finally, calculate g and δg with equations 24 and 25 and compare the accepted value.

6 Questions

1. Could the mass of the pulley be ignored and still yield decent results?
2. If this experiment were done on Venus, how would the rotational speed of the pulley (with the same masses) be affected? Explain.
3. Does friction affect the acceleration of the balls on the track? Explain.
4. In which of the experiments would you expect air resistance to play the smallest part? Explain.

7 Acceleration of Gravity Data Sheet

7.1 Atwood's Machine

$m_1 (g)$	$m_2 (g)$	$t_1 (s)$	$t_2 (s)$	$t_3 (s)$	$t_{avg} (s)$

7.2 Galileo's Ramp

H (cm)	S (cm)	d (cm)

Ball	$t_1 (s)$	$t_2 (s)$	$t_3 (s)$	$t_{avg} (s)$

H (cm)	S (cm)	d (cm)

Ball	$t_1 (s)$	$t_2 (s)$	$t_3 (s)$	$t_{avg} (s)$